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SYLLABUS

OF A

HUNDRED PHYSICAL MEASUREMENTS.

A TWO YEARS' COURSE

FOR THE USE OF THE

JEFFERSON PHYSICAL LABORATORY,

BY HAROLD WHITING, PH.D.,

Instructor in Physics at Harvard College.

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PREFACE.

THE first year's course of experiments is founded upon that originally given by Professor Trowbridge, with whose advice and assistance, together with that of Dr. Hall, the present system has been elaborated. To Dr. Hall thanks are also due for numerous corrections in the proof sheets.

Many of the experiments will be found described in Professor Trowbridge's "New Physics." From their greater educational value in the laboratory, those only of a quantitative nature have been selected, leaving all others to the lecture courses, in which they are amply exemplified.

The choice of experiments has been further modified by the present limited facilities of the Laboratory, and by the necessity of reduplicating apparatus in order that a large class may follow, practically, the same logical course of experiment.

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The text contains the directions heretofore given by dictation to the class, together with the outlines of the proofs of certain formulae, collected from various text-books, for the convenience of the student. Examples have been added to give definiteness to the work.

To make the course accessible to students not proficient in mathematics, tables have, in several cases, been introduced, facilitating the calculation of results without the use of formulae whose proof might be imperfectly understood. The student is asked to take these tables for granted as he would logarithms or trigonometric functions.

For the same reason, the formulae of the second year have been established without the use of the calculus.

Brief outlines of many experiments have been taken from Kohlrausch, Pickering and other standard authorities. Glazebrook and Shaw's manual appeared in time to furnish useful suggestions in reprinting the last few experiments.

The syllabus is evidently a hand-book rather than text-book, and considered separately from the course of lectures is necessarily incomplete.

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† Not required on account of insufficient apparatus.

‡ Omitted from lack of apparatus.

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VOLUME OF CYLINDER.

The Direct Measurement of the Volume of a Cylinder.

The length, l, of a steel cylinder* is measured by a vernier gauge, and the diameter, $2r$, by the micrometer gauge; the volume, v, is then calculated by the formula,

$$v = \pi r^2 l.$$

The micrometer gauge is applied by means of the friction head, and must be held so that the axis is perpendicular to that of the cylinder. The screw threads are very nearly a millimetre apart, and the distance is divided into hundredths by the graduation upon the head of the screw. It is well always to estimate the fraction of a division by

* This is the same cylinder whose displacement in grammes of water will afterwards be tested, so that the weight assumed for a cubic centimetre of water will be confirmed (as it was originally determined) in this way. The length and diameter will be required later on in magnetic experiments with the same cylinder, and should for this reason be preserved.

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the eye, even if the probable error be greater than a whole division.

The vernier gauge is graduated in millimetres, read by the arrow head; ten auxiliary marks upon the sliding piece constitute the *vernier*, and coincide successively with the main scale when the arrow is respectively 1, 2, 3, etc. tenths of a millimetre beyond a given division. When a lens is used, the coincidence will not usually be found to be exact for any particular mark upon the vernier, but the choice will lie between two, and in estimating by the eye the relative amounts of dislocation, the hundredths may obviously be supplied.

The vernier should be clamped, before reading, with a given pressure upon the cylinder, whose axis must be parallel to that of the guage, and in the use of both instruments, care must be taken to measure between surfaces which touch when the gauge is closed, and to subtract, algebraically, the zero reading, that is, the reading of the instrument when closed.

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If the ends of the cylinder are not flat, but conical, two-thirds of the measurements of length should be made at the edges*; and the diameter, which is of still greater importance, should be taken at about ten different distances, and in at least four different directions. The average diameter of the hole through the centre is found by measuring that of a plug which fits it; its length is equal nearly to the diameter of the cylinder.

In calculating the volume of the cylinder, both length and radius should be expressed in centimetres, and the volume of the hole must be subtracted, having been calculated in the same way.

If the gauges be of steel as well as the cylinder, since the former should be true at zero, the correction for temperature will be completely eliminated, and the result should be the volume or displacement of the cylinder at zero degrees, in cubic centimetres of space.

* For the measurement of the axis when the end is concave, see Trowbridge, *New Physics*, Experiment 42.

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EXAMPLE.

Distances from polished end, in cm.	Number of observations at equal angular intervals.	Mean reading of diameter, in cm.
0.2	6	1.2712
0.6	4	1.2707
1.0	4	1.2751
1.4	4	1.2740
1.8	8	1.2692
2.2	8	1.2697
2.6	4	1.2712
3.0	4	1.2738
3.4	4	1.2743
3.8	6	1.2710

Average diameter . . . 1.2720 cm.
" radius . . . 0.6360 cm.
Mean of 10 observations on edge 4.001 cm.
" " " near " 4.002 cm.
" " " at centre 4.012 cm.

Average length of cylinder . 4.005 cm.
Diameter of hole, mean of 10 observations . . . 0.310 cm.
Volume of completed cylinder,
$3.1416 \times 4.005 \times 0.6360^2 =$ 5.089 cu.cm.
Volume of hole,
$3.1416 \times 1.272 \times 0.155^2 =$ 0.096 cu. cm.

Actual volume of steel . . 4.993 cu. cm.

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CORRECTION OF GAUGES.

*The Correction of the Vernier and Micrometer Gauges by means of the Cathetometer.**

The distance between successive millimetre divisions upon the vernier gauge from 0 to 50 or upwards is accurately determined by means of the cathetometer, also that of each mark upon the vernier from the arrow head. The horizontal cross-hair of the telescope should divide the coarser ruling of the gauge into two sections of equal area, and the adjustment may be rapidly brought about by means of the screw within one or two thousandths of a centimetre.

The micrometer gauge is then held in a vise, the head set successively at every whole number of turns from 1 to 20, and the distance between the prongs is observed in each case.

* The Dividing Engine is sometimes used in place of the Cathetometer.

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Both gauges must be adjusted by the eye with the axis vertical, or what is more important, parallel to the shaft of the cathetometer, taking great care not to disturb the telescope in any way in the midst of a connected series of measurements.

The eye piece is once for all focussed upon the cross-hairs, so that these are distinctly visible; then the cross-hairs are made to coincide with the image of the object, until no *parallax* occurs when the eye is slightly raised or lowered.*

By obvious methods, the proper correction† is finally applied to the volume of the cylinder previously determined.

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* It is well to make sure that *both* ends of the scale are in focus before beginning the measurements.

† The correction for temperature may be neglected in this experiment.

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EXAMPLE.

Distances in cm. upon vernier gauge:—

Nominal.	Observed.	Nominal.	Observed.	Nominal.	Observed.	Nominal.	Observed.	Nominal.	Observed.
0.1	0.102	1.1	1.103	2.1	2.102	3.1	3.100	4.1	4.102
0.2	0.200	1.2	1.201	2.2	2.203	3.2	3.200	4.2	4.203
0.3	0.304	1.3	1.301	2.3	2.302	3.3	3.302	4.3	4.303
0.4	0.401	1.4	1.402	2.4	2.402	3.4	3.403	4.4	4.404
0.5	0.502	1.5	1.503	2.5	2.502	3.5	3.503	4.5	4.502
0.6	0.603	1.6	1.603	2.6	2.600	3.6	3.604	4.6	4.602
0.7	0.702	1.7	1.704	2.7	2.701	3.7	3.704	4.7	4.704
0.8	0.801	1.8	1.804	2.8	2.802	3.8	3.801	4.8	4.803
0.9	0.900	1.9	1.903	2.9	2.901	3.9	3.903	4.9	4.902
1.0	1.002	2.0	2.002	3.0	3.001	4.0	4.002	5.0	5.001

Distances of vernier divisions from arrow-head:—

Number	1	2	3	4	5	6	7	8	9	10
Nominal.	0.090	0.180	0.270	0.360	0.450	0.540	0.630	0.720	0.810	0.900
Observed.	0.093	0.182	0.271	0.362	0.451	0.542	0.633	0.726	0.816	0.904

Contact on micrometer gauge at +0.002 cm.

Distances between prongs, in centimetres:—

Nominal.	Observed.	Nominal.	Observed.	Nominal.	Observed.	Nominal.	Observed.
0.098	0.098	0.598	0.601	1.098	1.102	1.598	1.604
0.198	0.196	0.698	0.704	1.198	1.200	1.698	1.704
0.298	0.299	0.798	0.804	1.298	1.299	1.798	1.803
0.398	0.400	0.898	0.905	1.398	1.401	1.898	1.906
0.498	0.500	0.998	1.003	1.498	1.502	1.998	2.005

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Average length of cylinder from previous measurements . . .	4.005 cm.
Average position of coincidence (middle of vernier), (4.005 + 0.45) . . .	4.455 cm.
Correction for main scale at this point	+ 0.003 cm.
Average correction for vernier scale	+ 0.003 cm.
Corrected length of cylinder, (4.005 + .003 — .003) .	4.005 cm.
Diameter of cylinder from previous measurements . . .	1.2720 cm.
Correction for micrometer gauge	+ .0013 cm.
Corrected diameter . . .	1.2733 cm.
Corrected volume of completed cylinder	5.100 cu. cm.
Correction for diameter of hole .	+ .001 cm.
Corrected volume of hole, ($3.1416 \times 1.2733 \times .156^2$)	.097 cu. cm.
Corrected displacement of steel,	5.003 cu. cm.

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PITCH OF SCREW.

[Omitted in 1884-85.]

The Determination of the Pitch of a Screw.

I. The vernier of a cathetometer is made to advance one millimetre at a time by means of the micrometer screw, and the readings of the graduated head are accurately determined. The amount which the vernier advances for one revolution measures the average *pitch* of a given portion of a screw, and is easily calculated from the above data.

II. The screw of a dividing engine may be tested in a similar way, by making the cross-hair of the telescope coincide successively with the different divisions of a standard graduated scale.

In either case, the error of graduation may be eliminated by using different portions of the scale, and the pitch of the screw, being once determined, enables one to measure small lengths with great precision.

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THE SPHEROMETER.

The Measurement of the Curvature of a Lens by means of the Spherometer.

A spherometer with a millimetre thread is set upon plane glass, so that the least change in the central screw causes rocking to appear or to disappear. Numerous readings are obtained in this way, as well as upon the two opposite faces of a bi-convex lens, after which a second series should be made upon plane glass to eliminate the effects of wear, strain and temperature upon the instrument.

The distance through which the point has to be raised to adjust it to the curvature of the lens is to its mean oblique distance from the three feet of the spherometer, found by obvious measurements, as that distance is to the diameter of the sphere of which the given side of the lens is a part. The radius of curvature is of course equal to half the diameter, and will be of use, later on, in determining the refractive index of the glass.

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Example.

Mean of three settings on plane glass before setting on lens	1.3314 cm.
Ditto after setting on lens	1.3320 cm.
Average	1.3317 cm.
Mean of 3 settings on one face of lens,	1.1810 cm.
" " " other " " "	1.1812 cm.
Average	1.1811 cm.
Oblique distance from central point to first foot, 3 obs.	2.214 cm.
Ditto to second foot	2.203 cm.
Ditto to third foot	2.207 cm.
Average	2.208 cm.
Screw turned through distance, 1.3317 — 1.1811 = . . .	0.1506 cm.
Mean radius of curvature of lens, $\frac{1}{2} \times (2.208)^2 \div .1506 =$ . .	16.19 cm.

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NICHOLSON'S HYDROMETER.

The Determination of the Apparent Specific Gravity of a Solid by Means of Nicholson's Hydrometer.

The weight required to sink the hydrometer to a given mark is found first, without any other load; second, with a solid in the upper pan; and third, with the same solid in the lower pan. The difference of the first two weighings is the weight of the solid in air; that of the last two is the weight of water displaced; the quotient of the former weight by this displacement is the specific gravity at the given temperature.*

Since none of the weighings are made in vacuo, these results are only what are called *apparent*, including the specific gravity, which, however, if the solid be an alloy principally of lead, happens to agree very closely under ordinary conditions of the atmosphere with the corrected density at 0°.

* That is, the specific gravity referred to water of the same temperature.

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In this case, the corrections for temperature and buoyancy of air, which may always be calculated in precisely the same manner as with the hydrostatic balance, are not justified by the precision of the instrument.

The temperature should, however, be noted before and after the experiment, as the accuracy of the Method of Substitution depends upon its constancy; the instrument must not touch the sides of the containing vessel, and great care must be used to remove air bubbles which collect, especially in water freshly drawn, upon all the wetted surfaces. The stem should always be wet for a given distance—about one centimetre—above the water level, and then allowed to become sensibly dry before taking the final reading.

Should the upper pan or its contents by any accident be spattered or submerged, after a careful drying the experiment must be repeated from the beginning. The centigramme weights are the smallest which need be used, but by finding, once for all, the sensitiveness of the instrument to a change of one centigramme or decigramme, the number of milligrammes which would bring about the exact adjustment may be estimated by the eye.

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EXAMPLE.

Weight required to sink empty hydrometer nearly to given mark .	30.000g.
Correction for 2 mm. (at 4 mm. per centigramme)	.005 g.
	30.005 g.
Weight required with solid in upper pan	1.006 g.
Apparent weight of solid in air .	28.999 g.
Weight required to sink hydrometer to same mark with cylinder in lower pan	3.568 g.
Ditto in upper pan, brought down .	1.006
Displacement	2.562 g.
Apparent specific gravity at 17°, referred to water at 17°, $28.999 \div 2.562 =$ . . .	11.32

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THE BALANCE.

The Correction of a set of Weights.

[Omitted in 1884-1885.]

One of the weights in question is compared with a standard, and the others are referred to this one.

The balance is adjusted so that, when empty, the pointer oscillates about the central division, number 10 of the scale. The reading is always determined while the pointer is in motion, and with the case closed, to avoid currents of air, by noting the positions of an odd number of successive turning points, and taking the mean for each side, then adding and halving. The load should never be thrown violently on the knife edges, nor changed nor jarred in any way while the balance is swinging or free to swing, nor allowed to remain for an undue length of time; before leaving the balance, the arms should be

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properly supported, the weights returned to their places, to prevent loss or confusion*, and the case closed. Corrosive substances must never touch the pans, and both balance and weights must in every way be protected against dust, moisture and corrosion. With these precautions, the accuracy of the determinations which follow will be greatly increased.

The method of double weighing is always employed where accuracy, as well as precision, is required. In this way, by interchanging the load and weights, any error due to inequality of the arms or imperfect adjustment will be eliminated. In other cases, it must be specified which pan contains the weights.

The centigramme and decigramme weights are first balanced, one cg. weight against the other, both together against each of the 2 cg. weights, in turn, then the 5, 10, 20, 50 cg. weights are all balanced against their equivalents, so that they may be proved to be multiples of one of the cg. weights. Should any errors be noticed, they may be estimated in fractions of a centigramme

*The weights should be still further identified by a letter stamped upon them before beginning the experiment.

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by dividing the observed displacement of the pointer from the zero reading by that due to one centigramme, which is easily determined.

The gramme weight is then balanced against its equivalent in centigrammes, and the sum is expressed in terms of the gramme weight, which is taken as a temporary unit; one of the two 2 g. weights is then compared, and its value determined in units of the magnitude of the 1 g. weight; then, successively, all the other weights are determined in the same unit, up to the 100 g. weight, which is finally balanced against a standard of the same density, so that no correction for buoyancy of air will be necessary throughout the experiment. The quotient of this weight, in terms of the standard, by that in terms of the temporary unit is the constant by which all the weighings must be multiplied in order to find their true value. In the same way, the true value of the gramme weight being known, that of the small weights may, if necessary, be determined.

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EXAMPLE.

No error noticed in smaller weights.

100 CG = 1 G + 0.12 CG =	1.0012 g′ nearly
2 G = 100 CG + 1G + 0.13 CG =	2.0025 g′ "
2 G′ = 2 G =	2.0025 g′ "
5 G = 2 G + 2 G′ + 1 G + 0.10 CG =	5.0060 g′ "
10 G = 5 G + 2 G + 2 G′ + 1 G + 0.11 CG =	10.0121 g′ "
10 G′ = 10 G =	10.0121 g′ "
20 G = 10 G + 10 G′ =	20.0242 g′ "
50 G = 20 G + 10 G + 10 G′ + 5 G + 2 G + 2 G′ + 1 G + 0.21 CG =	50.0615 g′ "
100 G = 50 G + 20 G + 10 G + 10 G′ + 5 G + 2 G + 2 G′ + 1 G + 0.01 CG =	100.1210 g′ "

100 G = 100 g (standard) + 0.10 CG

= 100.0010 g.

*** g′ = (100.0010 ÷ 100.1210) = 0.998799 g.

100 CG =	1.0000 g.
1 G =	0.9988 g.
Hence 2 G =	2.0001 g.
2 G′ =	2.0001 g.
5 G =	5.0000 g.
10 G =	10.0001 g.
10 G′ =	10.0001 g.
20 G =	20.0002 g.
50 G =	50.0015 g.
100 G =	100.0010 g.

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HYDROSTATIC BALANCE.

The Determination of the Displacement and Density of a Solid by means of the Hydrostatic Balance.

By weighing a solid in air, and in water of known temperature, the displacement and density are determined.

The steel cylinder, whose volume has already been measured, is weighed carefully in air, then suspended by a fine wire from one arm of the balance, and weighed in a vessel of water which is supported by an arch so as not to rest upon the pan. The temperature of the water is accurately observed, and care must be taken to remove air bubbles with a fine brush, and to avoid capillary depression due to dryness of the wire. The weight of the latter is then separately found, under exactly the same conditions as before, and deducted from the previous result.

The brass weights displace very nearly as much air as the steel cylinder, hence the apparent weight of the latter is indistinguishable from its weight in vacuo; but the apparent weight in

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water is considerably too great, since there is nothing to balance the buoyancy of the air upon the brass weights, amounting to one milligramme for every seven grammes, in the ordinary condition of the atmosphere.

Subtracting the corrected weight in water from that in vacuo, we have the number of grammes of water displaced, and multiplying by the space occupied by each gramme at the temperature observed,* we find the volume of the cylinder, which may be reduced to zero, remembering that the change in one cubic centimetre for one degree is .000036. The result is then compared with that previously obtained by direct measurement.

Dividing the weight in vacuo by the volume at zero, we have the true density of the steel.

*SPACE OCCUPIED BY A GRAMME OF WATER IN CUBIC CENTIMETRES.

Temperature.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
15°	1.00083	84	86	87	89	91	92	94	95	97
16°	1.00099	100	102	104	105	107	109	111	112	114
17°	1.00115	117	119	121	123	125	126	128	130	132
18°	1.00133	135	137	139	141	143	145	147	149	151
19°	1.00153	155	157	159	161	163	165	167	169	171
20°	1.00173	175	177	179	181	184	186	188	190	192
21°	1.00194	196	199	201	203	205	207	210	212	214
22°	1.00217	219	221	223	226	228	230	232	235	237
23°	1.00239	242	244	246	249	251	253	256	258	260
24°	1.00263	265	267	270	272	275	277	279	282	284

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EXAMPLE.

Apparent weight of cylinder in air .	39.000 g.
" weight of cylinder in water with wire half in water. .	34.067 g.
Apparent weight of wire half in water .	.060 g.
" weight of cylinder in water	34.007 g.
Loss of 34 g. in air at 1 mg. per 7 g.	.005 g.
True weight of cylinder in water	34.002 g.
Weight of cylinder in vacuo .	39.000 g.
Weight of water displaced .	4.998 g.
1 g. of water at 16°.1 occupies	1.00100 cu. cm.
Volume of cylinder (1.001 × 4.998) . .	5.003 cu. cm.
Correction for 16°.1 (16.1 × 5.003 × .000036)	.003 cu. cm.
Volume at zero . . .	5.000 cu. cm.
Compare with volume by measurement with gauges . .	4.993 cu. cm.
Also with corrected value . .	5.003 cu. cm.
Density by hydrostatic balance (39. ÷ 5)	7.80 $\frac{g.}{cu.\ cm.}$

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CAPACITY OF VESSEL.

The Determination of the Capacity of a Specific Gravity Bottle.

Finding the weight of a bottle full of water, and subtracting that of the bottle containing only air, the capacity is first roughly determined, then corrected for the temperature of the water and for the weight of air.

The bottle is carefully cleaned, rinsed with alcohol, dried by the foot-blast, and alternately weighed and dried until a constant result is obtained; but care must be taken not to heat it in the hand, lest the air within should be considerably warmer than that without. It is then filled with water, dried with a cloth, and the temperature, which should be constant, is observed before and after weighing. The stopper must be well ground, and inserted with a given pressure, taking care not to enclose any air. Subtracting the weight of the bottle with air, from that with water, we have the apparent weight of the water contained.

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At the beginning and end of the experiment, the barometer, thermometer and hygrodeik should be consulted according to the special directions which accompany each instrument, so that the weight of dry air may be found in Table I, and corrected for moisture by Table II.

Both water and weights are buoyed up by the atmosphere in proportion to their respective bulks. The space occupied by the water is almost numerically equal to its apparent weight;* that taken up by the brass weights is between one-eighth and one-ninth as much,† and the difference, in cubic centimetres, multiplied by the weight of each cubic centimetre of air displaced, gives the total effect of buoyancy, that is, the correction to be added to the apparent weight of water in order to find the true weight in vacuo.

The corrected weight of water in grammes is multiplied by the number of cubic centimetres of space occupied by each gramme‡ to find the whole number of cubic centimetres, which is, finally, the *capacity* of the bottle.

---

* More exactly, at 15° add one-fifth, at 18° one-quarter, at 22° one-third of one per cent.

† The displacement of one gramme of brass or of German silver may be taken as 0.117 cubic centimetres.

‡ See Table in the experiment with the hydrostatic balance.

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EXAMPLE.

Weight of bottle, first drying . .	50.029 g.
" " " second " . . .	50.000
" " " third " . .	50.000
" " " with water . . .	129.754
Apparent weight of water . . .	79.754 g.
Capacity about $\frac{1}{3}$ per cent. greater,	80.0 cu. cm.
Displacement of brass weights (.117 × 79.75)	9.3 cu. cm.
Difference	70.7 cu. cm.
Weight of 1 cu. cm. of dry air at 20° and 76 cm.	.001204 g.
Correction for aqueous vapor, dew point + 6°	.000004
1 cu. cm. of atmosphere weighs . .	.001200 g.
Buoyancy of 70.7 cu. cm.	0.085 g.
Apparent w't of water, brought down .	79.754 g.
Weight of water in vacuo	79.839 g.
Space occupied by 1 g. of water at 21°.3	1.00201 cu. cm.
Capacity at 21°.3	80.000 cu. cm.

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I. WEIGHT IN GRAMMES OF ONE CUBIC CENTIMETRE OF DRY AIR. (KOHLRAUSCH.)

Barometric Pressure in Centimetres of Mercury. Latitude 45°.

Temperature.	72	73	74	75	76	77
15°	.001161	.001177	.001193	.001209	.001225	.001242
16°	.001157	.001173	.001189	.001205	.001221	.001237
17°	.001153	.001169	.001185	.001201	.001217	.001233
18°	.001149	.001165	.001181	.001197.	.001213	.001229
19°	.001145	.001161	.001177	.001193	.001209	.001224
20°	.001141	.001157	.001173	.001189	.001204	.001220
21°	.001137	.001153	.001169	.001185	.001200	.001216
22°	.001133	.001149	.001165	.001181	.001196	.001212
23°	.001130	.001145	.001161	.001177	.001192	.001208
24°	.001126	.001141	.001157	.001173	.001188	.001204
25°	.001122	.001138	.001153	.001169	.001184	.001200
26°	.001118	.001134	.001149	.001165	.001180	.001196
27°	.001114	.001130	.001145	.001161	.001176	.001192
28°	.001110	.001126	.001142	.001157	.001172	.001188
29°	.001107	.001122	.001138	.001153	.001169	.001184
30°	.001103	.001119	.001134	.001149	.001165	.001180

II. CORRECTION FOR MOISTURE.

To be applied to the weight of a cubic centimetre of air.

Dew Point	Subtract	Dew Point	Subtract	Dew Point	Subtract	Dew Point	Subtract
—10°	.000001	0°	.000003	+10°	.000006	+20°	.000010
— 8°	.000002	+2°	.000003	+12°	.000006	+22°	.000012
— 6°	.000002	+4°	.000004	+14°	.000007	+24°	.000013
— 4°	.000002	+6°	.000004	+16°	.000008	+26°	.000015
— 2°	.000003	+8°	.000005	+18°	.000009	+28°	.000016

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DENSITY OF FRAGMENTS.

The Determination of the Displacement and Density of a Solid by Means of the Specific Gravity Bottle.

A weighed quantity of a solid is introduced in fragments into the specific gravity bottle, and the capacity is re-determined. The difference is the displacement, and the quotient of the corrected weight in vacuo by the displacement is the density.

The solid should be insoluble in water, else some other liquid must be used, whose density, if unknown, may be determined later on. In all cases, the capacity is the product of the number of grammes and the space occupied by each gramme; and the latter is the reciprocal of the density.

The solid should nearly fill the specific gravity bottle, in which it is most conveniently weighed; it is then freed from air by exhaustion under the bell jar of an air pump, being barely

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covered with water, which, on account of ebullition, must not completely fill the bottle. The density of the water will not be materially affected by the process; it may, if necessary, be changed without exposing the solid to the air, in filling the bottle, previous to closing. Great care should be taken to prevent the bottle from touching any of the oily surfaces with which it is necessarily surrounded; in other respects the weighings and temperatures are found exactly as in the last experiment, and the difference between the old capacity and the new, corrected if necessary for the difference of temperature,* is the actual displacement of the solid.

Subtracting algebraically from this displacement that of the weight of the solid in brass,† and multiplying by the weight of a cubic centimetre of air,‡ we have the correction to be added to the apparent weight of the solid to find the true weight in vacuo; this, finally, divided by the displacement, is the required density at the given temperature.

---

* The capacity of a glass vessel increases by about one part in forty thousand for a rise of one degree in temperature.

† The displacement of one gramme of brass or German silver may be taken as 0.117 cubic centimetres.

‡ See Tables under Capacity of Vessel.

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Example.

Weight of bottle with air . .		50.000 g.
" " " " solid and air		214.963 g.
Apparent weight of solid . .		164.963 g.
Weight of bottle, solid and water .		244.871 g.
Subtract weight of bottle, solid and air		214.963 g.
Apparent weight of water at 21°.25		29.908 g.
Displacement of water, about $\frac{1}{8}$ per cent. higher		30.0 cu. cm.
Displacement of brass weights (29.94 × .117) . .		3.5 cu. cm.
Difference about . . .		26.5 cu. cm.
Buoyancy of 26.5 cu. cm. of air, density .0012 . .		.032 g.
Apparent weight of water, brought down . .	29.908 g.	
Weight of ditto in vacuo .	29.940 g.	
Space occupied by each gramme at 21°.25 .	1.002 cu. cm.	
New capacity of bottle at 21°.25	30.000 cu. cm.	
Former " " " reduced to 21°.25 . . .	80.000 cu. cm.	
Displacement of solid at 21°.25	50.000 cu. cm.	

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Displacement of solid, brought over	50.0 cu. cm.
Displacement of brass weights, (164.9 × .117) . .	19.3 cu. cm.
Difference	30.7 cu. cm.
Buoyancy of 30.7 cu. cm. of air, of density .0012 . .	0.037 g.
Weight of solid in vacuo (164.963 + 0.037) .	165.000 g.
Density at 21°.25 (165.000 ÷ 50.000) .	3.3 $\frac{\text{g.}}{\text{cu. cm.}}$

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DENSITY OF LIQUID.

The Determination of the Density and Strength of a Mixture of Alcohol and Water by Means of the Specific Gravity Bottle.

By weighing a bottle of known capacity, first empty, then filled with alcohol of a given temperature, the density and hence the strength of the latter is determined.

Having re-determined the weight of the bottle with air, the apparent weight of the diluted or commercial alcohol is found and corrected in precisely the same manner as that of the water in a previous experiment, remembering that the displacement of the alcohol is equal to the capacity* of the bottle. Dividing the weight in vacuo by the displacement, the density is found.

The temperature, which must lie between 15° and 22°, is observed with the utmost precision, both immediately before and immediately after weighing; the higher temperature is chosen, since the bottle can in neither case be more than full,

*The capacity of a glass vessel increases by one part in forty thousand for a rise in temperature of one degree centigrade.

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and selecting the two columns in the table* below, nearest that temperature, the percentage by weight† of absolute alcohol is calculated by double interpolation.

EXAMPLE.

Weight of bottle with alcohol	115.910	g.
" " " " air	50.000	g.
Apparent weight of alcohol	65.910	g.
Capacity of bottle	80.000	cu. cm.
Displacement of brass weights (65.9 × .117)	7.7	cu. cm.
Difference	72.3	cu. cm.
Buoyancy of 72.3 cu. cm. of air, density .00125	0.090	g.
Weight in vacuo (65.91 + .09)	66.000	g.
Displacement	80.000	cu. cm.
Density at 19°.5 (66 ÷ 80)	0.8250	$\frac{g.}{cu.\ cm.}$

* This table, whose probable error is about three ten-thousandths, is based upon the results of numerous early observers. See under *Alcohol, Eliot and Storer's Dictionary of Solubilities.*

† It is usual, in commerce, to reckon the per cents by volume; these are therefore compared in the table with the per cents by weight, at the ordinary temperature of 15°.

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Interpolated density for 87 per cent. alcohol at 19°.5 . . .	.82645
Ditto for 88 per cent. alcohol	.82385
Difference for 1 per cent. .	.00260
Difference of density of mixture from that of 87 per cent. (.82645—.825) . .	.00145
Per cent. by weight of absolute alcohol, $(87 + \frac{145}{260})$.	87.56%

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PER CENT. AND DENSITY OF DILUTE ALCOHOL.

w't	VOL. 15°	15°	16°	17°	18°	19°	20°	21°	22°
0	0.00	.9992	.9990	.9988	.9987	.9985	.9983	.9981	.9979
1	1.26	.9971	.9969	.9967	.9966	.9964	.9962	.9960	.9958
2	2.51	.9953	.9951	.9949	.9947	.9945	.9943	.9942	.9940
3	3.75	.9936	.9934	.9932	.9930	.9928	.9926	.9924	.9922
4	5.00	.9920	.9918	.9916	.9914	.9912	.9909	.9907	.9905
5	6.24	.9903	.9901	.9899	.9897	.9895	.9892	.9890	.9888
6	7.47	.9887	.9885	.9883	.9880	.9878	.9876	.9874	.9872
7	8.70	.9871	.9869	.9866	.9864	.9861	.9859	.9857	.9855
8	9.93	.9856	.9854	.9851	.9849	.9846	.9844	.9842	.9839
9	11.16	.9842	.9839	.9837	.9834	.9832	.9829	.9827	.9824
10	12.38	.9828	.9825	.9823	.9820	.9817	.9815	.9813	.9810
11	13.59	.9814	.9811	.9809	.9806	.9803	.9800	.9798	.9795
12	14.81	.9801	.9798	.9795	.9793	.9790	.9787	.9784	.9781
13	16.03	.9789	.9786	.9783	.9780	.9777	.9774	.9772	.9769
14	17.24	.9777	.9774	.9771	.9768	.9765	.9762	.9759	.9756
15	18.45	.9765	.9762	.9759	.9755	.9752	.9749	.9746	.9743
16	19.65	.9753	.9750	.9746	.9743	.9740	.9736	.9733	.9730
17	20.85	.9741	.9738	.9734	.9731	.9727	.9724	.9721	.9717
18	22.05	.9729	.9725	.9722	.9718	.9715	.9711	.9708	.9704
19	23.25	.9718	.9714	.9711	.9707	.9703	.9699	.9696	.9692
20	24.45	.9707	.9703	.9699	.9695	.9691	.9687	.9683	.9679
21	25.64	.9695	.9691	.9687	.9683	.9679	.9674	.9670	.9666
22	26.83	.9683	.9679	.9674	.9670	.9666	.9661	.9657	.9653
23	28.01	.9671	.9666	.9662	.9657	.9653	.9648	.9644	.9639
24	29.19	.9659	.9654	.9650	.9645	.9640	.9635	.9631	.9626
25	30.37	.9647	.9642	.9637	.9632	.9627	.9621	.9617	.9612
26	31.54	.9633	.9628	.9623	.9618	.9613	.9607	.9602	.9597
27	32.71	.9619	.9614	.9608	.9603	.9598	.9592	.9587	.9582
28	33.86	.9604	.9599	.9593	.9588	.9583	.9577	.9571	.9566
29	35.02	.9589	.9583	.9578	.9572	.9567	.9561	.9555	.9549
30	36.17	.9573	.9567	.9561	.9556	.9550	.9544	.9538	.9532
31	37.30	.9556	.9550	.9544	.9538	.9532	.9526	.9520	.9514
32	38.44	.9539	.9533	.9527	.9521	.9515	.9508	.9502	.9496
33	39.57	.9522	.9516	.9509	.9503	.9497	.9490	.9484	.9478
34	40.69	.9504	.9498	.9491	.9485	.9479	.9472	.9466	.9459
35	41.81	.9486	.9479	.9473	.9466	.9460	.9453	.9447	.9440
36	42.92	.9467	.9460	.9454	.9447	.9440	.9433	.9427	.9420
37	44.02	.9448	.9441	.9434	.9428	.9421	.9414	.9407	.9400
38	45.12	.9429	.9422	.9415	.9408	.9401	.9394	.9388	.9381
39	46.21	.9410	.9403	.9396	.9389	.9382	.9375	.9368	.9361
40	47.30	.9390	.9383	.9376	.9369	.9362	.9354	.9347	.9340
41	48.38	.9370	.9363	.9356	.9348	.9341	.9334	.9327	.9320
42	49.45	.9349	.9342	.9334	.9327	.9320	.9312	.9305	.9298
43	50.51	.9328	.9321	.9313	.9306	.9298	.9291	.9284	.9276
44	51.57	.9307	.9299	.9292	.9284	.9277	.9269	.9262	.9254
45	52.62	.9286	.9278	.9271	.9263	.9256	.9248	.9240	.9233
46	53.67	.9265	.9257	.9250	.9242	.9234	.9226	.9219	.9211
47	54.71	.9244	.9236	.9229	.9221	.9213	.9205	.9198	.9190
48	55.75	.9223	.9215	.9207	.9200	.9192	.9184	.9176	.9168
49	56.78	.9201	.9193	.9185	.9178	.9170	.9162	.9154	.9146
50	57.80	.9179	.9171	.9163	.9155	.9147	.9139	.9132	.9124

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W't	VOL. 15°	15°	16°	17°	18°	19°	20°	21°	22°
50	57.80	.9179	.9171	.9163	.9155	.9147	.9139	.9132	.9124
51	58.81	.9157	.9149	.9141	.9133	.9125	.9117	.9110	.9102
52	59.82	.9135	.9127	.9119	.9111	.9103	.9095	.9087	.9079
53	60.82	.9113	.9105	.9097	.9089	.9081	.9073	.9065	.9057
54	61.82	.9091	.9083	.9075	.9067	.9059	.9050	.9042	.9034
55	62.81	.9069	.9061	.9053	.9045	.9037	.9028	.9020	.9012
56	63.79	.9046	.9038	.9030	.9022	.9013	.9005	.8997	.8989
57	64.77	.9023	.9015	.9007	.8998	.8990	.8982	.8974	.8966
58	65.74	.9000	.8992	.8984	.8975	.8967	.8959	.8951	.8943
59	66.70	.8977	.8969	.8961	.8952	.8944	.8936	.8928	.8920
60	67.65	.8954	.8946	.8938	.8929	.8921	.8913	.8905	.8897
61	68.60	.8931	.8923	.8914	.8906	.8898	.8890	.8882	.8873
62	69.55	.8908	.8900	.8891	.8883	.8875	.8867	.8859	.8850
63	70.49	.8885	.8877	.8868	.8860	.8852	.8844	.8836	.8827
64	71.42	.8862	.8854	.8845	.8837	.8829	.8821	.8813	.8804
65	72.34	.8838	.8830	.8821	.8813	.8805	.8797	.8789	.8780
66	73.26	.8815	.8807	.8798	.8790	.8782	.8773	.8765	.8756
67	74.18	.8792	.8784	.8775	.8767	.8759	.8750	.8742	.8733
68	75.08	.8768	.8760	.8751	.8743	.8735	.8726	.8718	.8709
69	75.98	.8744	.8736	.8727	.8719	.8711	.8702	.8694	.8685
70	76.88	.8721	.8713	.8704	.8696	.8688	.8679	.8671	.8662
71	77.77	.8698	.8689	.8681	.8672	.8664	.8655	.8647	.8638
72	78.65	.8674	.8665	.8657	.8648	.8640	.8631	.8623	.8614
73	79.51	.8649	.8640	.8632	.8623	.8615	.8606	.8598	.8589
74	80.37	.8625	.8616	.8608	.8599	.8591	.8582	.8574	.8565
75	81.23	.8601	.8592	.8584	.8575	.8567	.8558	.8550	.8541
76	82.08	.8576	.8567	.8559	.8550	.8542	.8533	.8525	.8516
77	82.92	.8552	.8543	.8535	.8526	.8518	.8509	.8501	.8492
78	83.76	.8528	.8519	.8511	.8502	.8494	.8485	.8476	.8468
79	84.59	.8503	.8494	.8486	.8477	.8469	.8460	.8451	.8443
80	85.41	.8478	.8469	.8461	.8452	.8444	.8435	.8426	.8418
81	86.22	.8453	.8444	.8436	.8427	.8419	.8410	.8401	.8393
82	87.03	.8428	.8419	.8411	.8402	.8394	.8385	.8376	.8368
83	87.84	.8404	.8395	.8387	.8378	.8370	.8361	.8352	.8344
84	88.63	.8379	.8370	.8362	.8353	.8345	.8336	.8327	.8319
85	89.42	.8354	.8345	.8337	.8328	.8320	.8311	.8302	.8294
86	90.20	.8329	.8320	.8312	.8303	.8295	.8286	.8277	.8269
87	90.97	.8303	.8294	.8286	.8277	.8269	.8260	.8251	.8243
88	91.72	.8277	.8268	.8260	.8251	.8243	.8234	.8225	.8217
89	92.47	.8251	.8242	.8234	.8225	.8217	.8208	.8199	.8191
90	93.22	.8225	.8216	.8208	.8199	.8190	.8181	.8173	.8164
91	93.96	.8199	.8190	.8182	.8173	.8164	.8155	.8147	.8138
92	94.68	.8172	.8163	.8155	.8146	.8137	.8128	.8120	.8111
93	95.39	.8145	.8136	.8128	.8119	.8110	.8101	.8093	.8084
94	96.09	.8118	.8109	.8101	.8092	.8083	.8074	.8066	.8057
95	96.78	.8090	.8081	.8073	.8064	.8055	.8046	.8038	.8029
96	97.45	.8061	.8052	.8044	.8035	.8026	.8017	.8009	.8000
97	98.11	.8032	.8023	.8015	.8006	.7997	.7988	.7980	.7971
98	98.75	.8002	.7993	.7985	.7976	.7967	.7958	.7950	.7941
99	99.38	.7972	.7963	.7955	.7946	.7937	.7928	.7920	.7911
100	100.00	.7941	.7932	.7924	.7915	.7906	.7897	.7889	.7880

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THE DENSIMETER.

[Not required in 1884-85.]

The Calibration and Use of the Densimeter.

An ordinary specific gravity hydrometer is tested in water and at least one other liquid of known density and temperature, and the graduation is examined by the cathetometer. A table of corrections is made out, and numerous densities are rapidly determined.

The stem of the instrument must be of sensibly uniform calibre, and the distances of the divisions, both from the water-mark and from each other, should for a given instrument,* bear a constant ratio to the corresponding distances in the table below. Having once determined this ratio by testing in two liquids, and measuring the distance between the marks nearest the surface in each case, the distances of all the principal divisions are easily reduced to the scale upon which the table is constructed, and the densities indicated are corrected by means of the true values which that table contains.

* Neglecting the corrections for the buoyancy of air.

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HYDROMETRIC TABLE.

Showing the distance which the water-mark should be above or below the free surface of a liquid of given density in the case of a hydrometer in the shape of a uniform straight rod which floats at unit depth in water of maximum density.

Density.	Distance.	Difference.	Density.	Distance.	Difference.	Density.	Distance.	Difference.	Density.	Distance.	Difference.
0.70	.4286	201	1.00	.0000	99	1.30	.2308	58	1.60	.3750	39
0.71	.4085	196	1.01	.0099	97	1.31	.2366	58	1.61	.3789	38
0.72	.3889	190	1.02	.0196	95	1.32	.2424	57	1.62	.3827	38
0.73	.3699	185	1.03	.0291	94	1.33	.2481	56	1.63	.3865	37
0.74	.3514	181	1.04	.0385	91	1.34	.2537	56	1.64	.3902	37
0.75	.3333	175	1.05	.0476	90	1.35	.2593	54	1.65	.3939	37
0.76	.3158	171	1.06	.0566	88	1.36	.2647	54	1.66	.3976	36
0.77	.2987	166	1.07	.0654	87	1.37	.2701	53	1.67	.4012	36
0.78	.2821	163	1.08	.0741	85	1.38	.2754	52	1.68	.4048	35
0.79	.2658	158	1.09	.0826	83	1.39	.2806	51	1.69	.4083	35
0.80	.2500	154	1.10	.0909	82	1.40	.2857	51	1.70	.4118	34
0.81	.2346	151	1.11	.0991	80	1.41	.2908	50	1.71	.4152	34
0.82	.2195	147	1.12	.1071	79	1.42	.2958	49	1.72	.4186	34
0.83	.2048	143	1.13	.1150	78	1.43	.3007	49	1.73	.4220	34
0.84	.1905	140	1.14	.1228	76	1.44	.3056	47	1.74	.4253	33
0.85	.1765	137	1.15	.1304	75	1.45	.3103	48	1.75	.4286	33
0.86	.1628	134	1.16	.1379	74	1.46	.3151	46	1.76	.4318	32
0.87	.1494	130	1.17	.1453	72	1.47	.3197	46	1.77	.4350	32
0.88	.1364	128	1.18	.1525	72	1.48	.3243	46	1.78	.4382	32
0.89	.1236	125	1.19	.1597	70	1.49	.3289	44	1.79	.4413	31
0.90	.1111	122	1.20	.1667	69	1.50	.3333	45	1.80	.4444	31
0.91	.0989	119	1.21	.1736	67	1.51	.3377	44	1.81	.4475	31
0.92	.0870	117	1.22	.1803	67	1.52	.3421	44	1.82	.4505	30
0.93	.0753	115	1.23	.1870	65	1.53	.3464	42	1.83	.4536	31
0.94	.0638	112	1.24	.1935	65	1.54	.3506	42	1.84	.4565	29
0.95	.0526	109	1.25	.2000	63	1.55	.3548	42	1.85	.4595	30
0.96	.0417	108	1.26	.2063	63	1.56	.3590	41	1.86	.4624	29
0.97	.0309	105	1.27	.2126	62	1.57	.3631	40	1.87	.4652	28
0.98	.0204	103	1.28	.2188	60	1.58	.3671	40	1.88	.4681	29
0.99	.0101	101	1.29	.2248	60	1.59	.3711	40	1.89	.4709	28
1.00	.0000		1.30	.2308		1.60	.3750	39	1.90	.4737	28

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In reading the instrument, the stem of which should first be allowed to dry, the level of the under surface of the liquid should always be sighted, and a small movable ring is used as a mark in estimating the tenths of the divisions.

Example.

Reading of hydrometer in water at 19°	0.9993
Density of ditto . . .	0.9985
Correction . . .	—.0008
Reading of hydrometer in impure alcohol at 17° . . .	0.8225
Density by sp. gr. bottle reduced to 17°	0.8242
Correction	+ .0017
Corrected density at water mark . .	.9992
" " " mark .8220 .	.8237
Theoretical distance, according to table, between density 1. and density .9992 . . .	.0008
Ditto between density 1. and .8237,	.2141
Ditto between density .9992 and .8237, .2141 — .0008 = . . .	.2133
Distance measured by cathetometer	7.730cm.
Constant ratio .2133÷7.730 = .	.0276

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Density indicated.	Distance in cm. from water-mark, by cathetometer.	Ditto × .0276 = theoretical distance from water-mark.	Ditto + .0008 = theoretical distance from density 1.	Ditto corresponds to true density in table.	Correction = true density less density indicated.
1.00	0.000	.0000	.0008	.9992	—.0008
0.95	1.866	.0515	.0523	.9503	+.0003
0.90	3.937	.1087	.1095	.9013	+.0013
0.85	6.214	.1715	.1723	.8531	+.0031
0.80	8.772	.2421	.2429	.8046	+.0046
0.75	11.658	.3218	.3226	.7561	+.0061

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THE AIR-PUMP.

[Not required in 1884-85.]

The Determination of the Density of Air by means of a Partial Vacuum.

A weighed vessel is nearly exhausted of air by the air-pump, the loss of weight being carefully determined, then opened under water; the latter replacing bulk for bulk that portion of the air which has been exhausted, the density is determined in the usual way

A bottle is selected with a carefully ground glass stopper, slightly oiled to prevent leakage, and an exact weighing is made, care being taken not to warm the bottle by the hand. The temperature of the air is then found within a tenth of a degree, and the barometer read to hundredths of an inch, and reduced to centimetres by means of the table below The stopper is loosened before placing the bottle under the bell-jar of the air-pump, and after exhausting as much of the air as possible, the loss of weight is determined with the utmost precision Immediately the bottle is plunged into a tank of water, then opened and corked again, somewhat below the surface The

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gain of weight, determined by an ordinary balance, gives the effective capacity of the bottle, and the density of the atmosphere is the quotient of the former loss of weight by this effective capacity.*

The result is compared with the tables for the given temperature and pressure, and the degree of exhaustion is found by dividing the effective by the actual capacity of the bottle.

EXAMPLE.

Weight of bottle with air . .	150.304 g.
" " " after exhaustion	150.000 g.
" " air exhausted . .	.304 g.
" after admitting water .	400.00 g.
Subtract empty weight, about	150.00 g.
Weight of water . . .	250.00 g.
Volume of 1 gramme of water at 15°	1.0009 cu.cm.
Effective capacity . . .	250.23 cu.cm.
Actual capacity . . .	263.00 cu.cm.
Degree of exhaustion (250.23 ÷ 263)	95%
Tabulated density of atmosphere for 17° and 76.0 cm. .	.001217 $\frac{g.}{cu.\ cm.}$
Observed density (0.304 ÷ 250.23) . .	.001215 $\frac{g.}{cu.\ cm.}$

* This quotient will not be affected by the buoyancy of air upon the brass weights, which may therefore be neglected in this experiment.

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BAROMETRIC TABLE.

Reduction of Inches to Centimetres.

Inches.	0	1	2	3	4	5	6	7	8	9
28.0	71.119	.145	.170	.196	.221	.246	.272	.297	.323	.348
28.1	71.373	.399	.424	.450	.475	.500	.526	.551	.577	.602
28.2	71.627	.653	.678	.704	.729	.754	.780	.805	.831	.856
28.3	71.881	.907	.932	.958	.983	*008	*034	*059	*085	*110
28.4	72.135	.161	.186	.212	.237	.262	.288	.313	.339	.364
28.5	72.389	.415	.440	.466	.491	.516	.542	.567	.593	.618
28.6	72.643	.669	.694	.720	.745	.770	796	.821	.847	.872
28.7	72.897	.923	.948	.974	.999	*024	*050	*075	*101	*126
28.8	73.151	.177	.202	.228	.253	.278	.304	.329	.355	.380
28.9	73.405	.431	.456	.482	.507	.532	.558	.583	.609	.634
29.0	73.659	.685	.710	.736	.761	.786	.812	.837	.863	.888
29.1	73.913	.939	.964	.990	*015	*040	*066	*091	*117	*142
29.2	74.167	.193	.218	.244	.269	.294	.320	.345	.371	.396
29.3	74.421	.447	.472	.498	.523	.548	.574	.599	.625	.650
29.4	74.675	.701	.726	.752	.777	.802	.828	.853	.879	.904
29.5	74.929	.955	.980	*006	*031	*056	*082	*107	*133	*158
29.6	75.183	.209	.234	.260	.285	.310	.336	.361	.387	.412
29.7	75.437	.463	.488	.514	.539	.564	.590	.615	.641	.666
29.8	75.691	.717	.742	.768	.793	.818	.844	.869	.895	.920
29.9	75.945	.971	.996	*022	*047	*072	*098	*123	*149	*174
30.0	76.199	.225	.250	.276	.301	.326	.352	.377	.403	.428
30.1	76.453	.479	.504	.530	.555	.580	.606	.631	.657	.682
30.2	76.707	.733	.758	.784	.809	.834	.860	.885	.911	.936
30.3	76.961	.987	*012	*038	*063	*088	*114	*139	*165	*190
30.4	77.215	.241	.266	.292	.317	.342	.368	.393	.419	.444
30.5	77.469	.495	.520	.546	.571	.596	.622	.647	.673	.698
30.6	77.723	.749	.774	.800	.825	.850	.876	.901	.927	.952
30.7	77.977	*003	*028	*053	*079	*104	*130	*155	*180	*206
30.8	78.231	*257	.282	.307	.333	.358	.384	.409	.434	.460
30.9	78.485	.511	.536	.561	.587	.612	.638	.663	.688	.714
31.0	78.739	.765	.790	.815	.841	.866	.892	.917	.942	.968
31.1	78.993	*019	*044	*069	*095	*120	*146	*171	*196	*222
31.2	79.247	.273	.298	.323	.349	.374	.400	.425	.450	.476
31.3	79.501	.527	.552	.577	.603	.628	.654	.679	.704	.730
31.4	79.755	.781	.806	.831	.857	.882	.908	.933	.958	.984
31.5	80.009	.035	.060	.085	.111	.136	.162	.187	.212	.238
DIF. In.	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010
DIF. Cm.	.003	.005	.008	.010	.013	.015	.018	.020	.023	.025

* The star indicates that the number of whole centimetres is to be read from the line underneath it.

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DENSITY OF DRY AIR AT 0°

[Omitted in 1884–85.]

The Determination of the Density of Dry Air at 0° by means of a Sprengel Pump.

A glass receiver is completely exhausted of air by means of a Sprengel pump, and weighed with a counterpoise of equal displacement; it is then surrounded with melting ice, and dry air is admitted through a tube filled with chloride of calcium, until perfect equilibrium is established. The receiver is then hermetically closed and re-weighed after attaining the temperature of the room.

The difference in weight divided by the capacity at 0°, determined and corrected as in experiments with the specific gravity bottle, gives the density of dry air at this temperature, under the observed pressure of the barometer, which is reduced to 76 cm. by direct proportion. Under these conditions, for the latitude of Cambridge, the weight of a cubic centimetre of dry air should be about 0.0012925 grammes.

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DENSITY OF AIR.

The Determination of the Density of Dry Air by means of a Water Vacuum.

By condensing steam in a flask, a vacuum is obtained. The admission of air increases the weight of the flask by an amount easily determined with a delicate balance; this, divided by the additional weight of water necessary to fill the flask, gives the relative density of air and water.

The steam is generated in a four-ounce Florence flask, one-quarter full of water, resting in a sand bath, and a soft rubber stopper, slightly oiled, is dropped loosely into place. When all the air has been driven out, the heat is cut off, and the flask, covered with a towel, is removed and corked just before the ebullition has ceased, guarding carefully against explosion. The neck is plunged immediately under water, and the stopper driven firmly into place. The contents are gradually cooled by immersion, avoiding sudden and local changes of temperature, and the flask in an inverted position is thoroughly cleaned and dried. Leakage is shown by the entrance of air-bubbles, the absence of which is not, however, sufficient proof of the preservation of the

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vacuum. The sharp ring of the water-hammer and occasional violence of ebullition, in cooling, alone justify the continuation of the experiment.

The flask is then weighed rapidly, but with the utmost precision, and re-weighed with equal care after admitting the outside air through a narrow orifice. The stopper must be inserted up to a given mark in both weighings, and care must be taken not to warm the flask by the hand, until the temperature of the contents has been taken. The pressure of the atmosphere is found from the reading of the barometer, and the dew-point, nearly enough, from the hygrodeik. The flask is then filled with water, whose temperature should be noted, and, the stopper being properly inserted, the new weight is determined by an ordinary balance.

The capacity for air in the experiment is found by multiplying together the additional weight of water and the space occupied by each gramme;* then multiplying by the density of aqueous vapor corresponding to the atmospheric dew-point,† observed, we have the weight of this vapor carried in with the air, which must be of course subtracted. Dividing the weight of air, thus corrected, by the space which it occupies, we

* See Table under Hydrostatic Balance.

† See Table, below.

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have the density of dry air, which is to be compared with that tabulated for the given temperature and pressure.* It must be remembered that the air proper exerts only that portion of the total pressure, within the flask, which remains after deducting the tension of the aqueous vapor with which it is saturated. Taking for the dew-point the temperature of the water beneath the air, this tension may be found from the table below.

* See under Capacity of Vessel. No account need be taken of the buoyancy of the air upon the brass weights in this experiment, since it is eliminated in the quotient.

TENSION AND DENSITY OF AQUEOUS VAPOR. (KOHLRAUSCH.)

Dew-point.	Tension in cm. of mercury.	Density.	Dew-point.	Tension in cm. of mercury.	Density.
—10°	0.20	.0000021	+10°	0.91	.0000094
— 9	0.22	.0000024	11	0.98	.0000100
— 8	0.24	.0000027	12	1.04	.0000106
— 7	0.26	.0000030	13	1.11	.0000113
— 6	0.28	.0000032	14	1.19	.0000120
— 5	0.31	.0000035	15	1.27	.0000128
— 4	0.33	.0000038	16	1.35	.0000136
— 3	0.36	.0000041	17	1.44	.0000145
— 2	0.39	.0000044	18	1.54	.0000151
— 1	0.42	.0000046	19	1.63	.0000162
0	0.46	.0000049	20	1.74	.0000172
+ 1	0.49	.0000052	21	1.85	.0000182
2	0.53	.0000056	22	1.97	.0000193
3	0.57	.0000060	23	2.09	.0000204
4	0.61	.0000064	24	2.22	.0000215
5	0.65	.0000068	25	2.36	.0000229
6	0.70	.0000073	26	2.50	.0000242
7	0.75	.0000077	27	2.65	.0000256
8	0.80	.0000081	28	2.81	.0000270
9	0.85	.0000088	29	2.98	.0000286
+10°	0.91	.0000094	30°	3.16	.0000301

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EXAMPLE.

Weight of flask before admitting air	50.000 g.
" " " after " "	50.141 g.
" " flask full of water . .	170.0 g.
" " water added (170 — 50) .	120.0 g.
Volume of 1 gramme of water at 20°	1.0017 cu.cm.
Capacity for air (120 × 1.0017)	120.2 cu.cm.
Density of aqueous vapor at + 8°	.0000081 $\frac{g.}{cu.\ cm.}$
Weight " " " carried into flask (120.2 × .0000081) .	.001 g.
Weight of atmospheric air admitted, (50.141 — 50.000) . . .	.141 g.
Weight of dry air in flask . . .	.140 g.
Reading of barometer . . .	75.74 cm.
Tension of aqueous vapor in flask at 20°	1.74 cm.
Pressure of air proper . . .	74.00 cm.
Density tabulated for dry air at 20° and 74.00 cm. . . .	0.001173 $\frac{g.}{cu.\ cm.}$
Density observed (0.140 ÷ 120.2)	0.001165 $\frac{g.}{cu.\ cm.}$

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DENSITY OF GASES.

The Determination of the Density of a Gas by means of the Specific Gravity Flask.

A flask filled successively with air, coal-gas, and water, is carefully weighed in each case; the density of air and water being known, that of the coal-gas is determined.

The largest and lightest possible flask is selected, and being filled with dry air, is corked and weighed with the greatest precision.* The air is then replaced by coal-gas, by upward displacement, and the flask, being corked carefully so as not to admit air, is re-weighed with the same exactness. In each case the stopper is inserted to a given mark, and the temperature is observed with a delicate thermometer. The barometer is read to hundredths of an inch.

* As in the Density of Air, it is unnecessary to consider the buoyancy of the atmosphere upon the brass weights.

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The flask is then filled with water of known temperature, corked and weighed on an ordinary balance. The capacity of the flask is calculated, and the weight of air it contains is found for the given temperature and pressure.* This weight enables us to calculate that of the empty flask, and hence the weight of the coal-gas. Dividing the latter by the capacity, we have the density for the given temperature and pressure. Finally this density is reduced to 0° and 76 cm. pressure by the ordinary formula.

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* See under Capacity of Vessel.

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Example.

Weight of flask with water . .	270.0 g.
" " " " air . .	30.0 g.
" in air of water . . .	240.0 g.
Capacity of flask, about $\frac{3}{4}$ % higher,	240.7 cu. cm.
Weight of 1 cu. cm. of air at 20° and 75 cm. pressure . .	0.001189 g.
Exact weight of air in flask (240.7 × .001189) . .	0.286 g.
Exact weight of flask with air . .	30.000 g.
" " empty flask . . .	29.714 g.
" " flask with coal-gas . .	29.842 g.
Weight of coal-gas	0.128 g.
Density of coal-gas at 20° and 75 cm. (0.128 ÷ 240.7) . .	.000532 $\frac{g.}{cu.\ cm.}$

Ditto reduced to 0° and 76 cm. pressure.

$$.000532 \times \frac{293}{273} \times \frac{76}{75} = .000578 \ \frac{g.}{cu.\ cm.}$$

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VAPOR DENSITY.

[Omitted in 1884-85.]

The Determination of the Density and Tension of Vapors by Gay-Lussac's Method.

A weighed quantity of a liquid is introduced into a barometer or into the short arm of a Mariotte's tube, from which all air has been exhausted, and the tension is found by subtracting from the atmospheric pressure or adding to it, as the case may be, the pressure due to the difference of the two mercury levels. If a proper quantity of the liquid be used, the whole may be made to evaporate by lowering the level of the mercury in the tube, which must be carefully calibrated before beginning the experiment, and dividing the weight by the space occupied, we may determine the density of either the saturated or the rarified vapor, according to the degree of expansion allowed.

The experiment can be repeated at different temperatures by surrounding the tubes with the necessary heating or cooling apparatus.*

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*See Cooke, NEW CHEMISTRY, Lecture III, "Hofmann's method."

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DUMAS' METHOD

[Omitted in 1884-85.]

*The Determination of the Density of Saturated or Superheated Vapors by Dumas' Method.**

A liquid is boiled in a light glass globe, surrounded with water or paraffine, and sealed either when the last drop of liquid is disappearing or when a definite temperature above the boiling point has been reached. Knowing the empty weight and capacity of the globe, the density of the saturated or superheated vapor is determined as with the specific gravity flask.

The principle object of the determination of vapor densities is to find the specific gravity referred to hydrogen under the same conditions of temperature and pressure, which determines the proportions which enter into chemical combination, and is, according to the kinetic theory, equal to the ratio of the molecular weights.

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* See Cooke, NEW CHEMISTRY, Lecture III. "Dumas' method."

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SIMPLE PENDULUM.

The Determination of the Acceleration of Gravity by Means of the Simple Pendulum.

A small sphere of known diameter is suspended by a fine thread of measured length, and its time of oscillation noted; from these data the acceleration of gravity is determined.

The thread is of waxed silk, and is suspended from the sharp projecting edge of a firm bracket, along which it is prevented from sliding by a sharp groove. The under surface of the bracket must be horizontal, and must contain the so-called centre of suspension. The ball of the pendulum is a one-ounce leaden bullet, grooved with a knife so as to be held in a loop of the thread. The distance between the lower edge of the bracket and the top of the bullet is made one metre by the rod as nearly as possible; and adding one-half the average diameter of the bullet, found by the vernier gauge, we have practically the length of the pendulum.

The oscillations must now be watched for at least twenty minutes in succession, noting each time, as closely as possible, when the seconds clock seems to tick exactly as the pendulum is passing the middle point of its swing—marked

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by a plumb line—and these times may be read from any well regulated watch or clock.

Skipping a sixth of the coincidences, one is selected, and corrected by averaging with it all that precede, together with an equal number that follow. Another is selected, similarly, near the end of the series, and corrected in the same way. Each period of coincidence between these two indicates that the pendulum has made, in that period, one less swing than the clock. Knowing the rate of the clock, the time of vibration of the pendulum is easily calculated.

Finally, making sure that the length of the pendulum is unchanged, the acceleration of gravity is found by the table below.

TIME OF OSCILLATION.

Length of pendulum.								
99.0	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975	0.9970	0.9965
99.1	1.0006	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975	0.9970
99.2	1.0011	1.0005	1.0000	0.9995	0.9990	0.9985	0.9980	0.9975
99.3	1.0016	1.0011	1.0005	1.0000	0.9995	0.9990	0.9985	0.9980
99.4	1.0021	1.0016	1.0010	1.0005	1.0000	0.9995	0.9990	0.9985
99.5	1.0026	1.0021	1.0015	1.0010	1.0005	1.0000	0.9995	0.9990
99.6	1.0031	1.0026	1.0020	1.0015	1.0010	1.0005	1.0000	0.9995
99.7	1.0036	1.0031	1.0026	1.0020	1.0015	1.0010	1.0005	1.0000
99.8	1.0041	1.0036	1.0031	1.0025	1.0020	1.0015	1.0010	1.0005
99.9	1.0046	1.0041	1.0036	1.0030	1.0025	1.0020	1.0015	1.0010
100.0	1.0051	1.0046	1.0041	1.0035	1.0030	1.0025	1.0020	1.0015
100.1	1.0056	1.0051	1.0046	1.0040	1.0035	1.0030	1.0025	1.0020
100.2	1.0061	1.0056	1.0051	1.0045	1.0040	1.0035	1.0030	1.0025
100.3	1.0066	1.0061	1.0056	1.0050	1.0045	1.0040	1.0035	1.0030
100.4	1.0071	1.0066	1.0061	1.0056	1.0050	1.0045	1.0040	1.0035
100.5	1.0076	1.0071	1.0066	1.0061	1.0055	1.0050	1.0045	1.0040
100.6	1.0081	1.0076	1.0071	1.0066	1.0060	1.0055	1.0050	1.0045
100.7	1.0086	1.0081	1.0076	1.0071	1.0065	1.0060	1.0055	1.0050
100.8	1.0091	1.0086	1.0081	1.0076	1.0070	1.0065	1.0060	1.0055
100.9	1.0096	1.0091	1.0086	1.0081	1.0075	1.0070	1.0065	1.0060
101.0	1.0101	1.0096	1.0091	1.0086	1.0080	1.0075	1.0070	1.0065
g =	977	978	979	980	981	982	983	984

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EXAMPLE.

Distance from lower surface of bracket to top of ball	100.02
Semi-diameter of ball, mean of 10 observations	0.98
Length of pendulum . . .	101.00

Times of coincidence by watch.

1	13m. 41s.	6	24m. 5s.	11	34m. 32s.
2	15m. 44s.	7	26m. 10s.	12	36m. 34s.
3	17m. 51s.	8	28m. 19s.	13	38m. 39s.
4	19m. 53s.	9	30m. 21s.	14	40m. 46s.
5	22m. 1s.	10	32m. 25s.	15	42m. 49s.
3	17m. 50s.	AVERAGE.		13	38m. 40s.

Corrected time of 3d coincidence	17m. 50s.
" " " 13th coincidence	38m. 40s.
Period of 10 coincidences . .	20m. 50s.
Average time between coincidences .	125s.
Time of swing of seconds clock . .	.99993s.

Time of pendulum:

$$\frac{125}{124} \times .99993 = 1.0080$$

Acceleration of gravity from table . 981.

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KATER'S PENDULUM.

[Not required in 1884-85.]

The Determination of the Acceleration of Gravity by means of Kater's Pendulum.

Two movable knife-edges are arranged upon the shaft of the pendulum so counterpoised that it vibrates in the same time upon either of them; the intervening distance is then equal to the length of the equivalent simple pendulum.

One knife-edge is clamped near the end of the rod, the other at a distance (l) measured by the verniers, as nearly as possible equal to one metre. A counterpoise is set by trial so that the time of oscillation in a small arc of less than one degree is nearly half a second upon either knife-edge, as determined by the method of coincidences used with the simple pendulum. The mean time of oscillation being t, the acceleration of gravity is calculated by the formula,

$$g = \frac{\pi^2 l}{t^2},$$

The proof of the formula for Kater's pendulum may be divided into three propositions: 1st, that it holds for the conical pendulum; 2d, that the conical pendulum is synchronous with the simple pendulum; and 3d, that the distance

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between the knife edges is really the length of the equivalent simple pendulum.

1.* In the conical pendulum we have two forces in equilibrium, one centrifugal, the other centripetal. If m is the mass, r and v the instantaneous radius and velocity of revolution, and t the time between successive elongations, the centrifugal force, F, is $\frac{mv^2}{r}$, or substituting for v its value, $\frac{2\pi r}{2t}$, $F = \frac{m}{r}\left(\frac{\pi^2 r^2}{t^2}\right) = \frac{m\pi^2 r}{t^2}$.

Now the ratio of the horizontal component of the centripetal force, P, to the vertical force of gravity, mg, will equal the ratio of the horizontal displacement, r, to the vertical length, l, of the pendulum, that is, $P = \frac{r}{l} \times mg$.

Hence we have $F = P = \frac{m\pi^2 r}{t^2} = \frac{rmg}{l}$,

which gives† $g = \frac{\pi^2 l}{t^2}$.

* See Trowbridge, New Physics, Experiments 77 and 78.

† If the displacement of the pendulum be less than one-hundredth of its length, the true length of the pendulum will not differ from its vertical component by one part in twenty thousand, which, therefore, represents the error of the formula in this case.

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2. Conical motion may be decomposed into two motions in vertical planes at right angles, and the components of the forces in one plane can have no effect upon the motion in the other. Confining our attention to the motion in either plane, we see that the component of centripetal force is the same for a given small displacement from the vertical as in the case of a simple pendulum.* (See above.) Hence the components of acceleration and of velocity must be the same, point for point, as for the simple pendulum, whose time of oscillation must be approximately† expressed by the formula,

$$t = \pi \sqrt{\frac{l}{g}}.$$

3. Multiplying both numerator and denominator under the radical by ml, we have‡

$$t = \pi \sqrt{\frac{ml^2}{mlg}}.$$

* The simple pendulum with its system of forces and accelerations, is most simply represented by the shadow of the conical system upon the wall.

† Strictly, however, as the directive force diminishes in the ratio of the sine to the arc of the angle of displacement, an error is introduced opposite in sign, but neccessarily less in amount than in the case of the conical pendulum; for, even if the directive force had its mimimum value throughout the whole swing, this error would not exceed one-third of the above in the given case, and for small arcs, in general, will be found to be about four times less in the case of the simple pendulum, both errors varying very nearly as the square of the amplitude of oscillation.

‡ See Trowbridge, New Physics, Experiment 81 *et seq.*

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Writing K in place of ml^2, the moment of inertia, and D in place of mlg, the directive force, we have

$$t = \pi \sqrt{\frac{K}{D}};$$

a formula which must hold also for the compound pendulum. But the moment of inertia of the latter is evidently the sum of two parts, one due to the direct displacement of the centre of gravity, the other to rotation about it. Letting M be the mass, L, the length of the equivalent simple pendulum divided into two parts, namely x, the distance from the centre of suspension to the centre of gravity, and y, the distance from the centre of gravity to the centre of oscillation, and let k be the radius of gyration about the centre of gravity, defined so that Mk^2 is equal to the moment of inertia about this centre, we have

$$K = Mx^2 + Mk^2$$
$$D = Mxg$$

Hence

$$t = \pi \sqrt{\frac{Mx^2 + Mk^2}{Mxg}} = \pi \sqrt{\frac{x^2 + k^2}{xg}} = \pi \sqrt{\frac{L}{g}},$$

the latter by definition of L, and we see that

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$$L = \frac{x^2 + k^2}{x} = x + \frac{k^2}{x}, \text{ whence}$$

$$k^2 = (L - x)x = yx;$$

that is, the product of the two segments made by the centre of gravity is constant and equal to the square of the radius of gyration about this centre.

Interchanging the centres of suspension and oscillation, we have new segments as follows: $x' = y$, and therefore since $x'y' = k^2 = xy$, $y' = x$, whence $x' + y' = L' = x + y = L$, and therefore the time of oscillation will be the same as before.

There are evidently four and only four points which satisfy this condition, two at a distance x, one on each side of the centre of gravity, and two at a distance y. If care is taken to find a pair of points of suspension, on opposite sides of the centre of gravity and at unequal distances from it which give the same (or very nearly the same) time of oscillation, we have a right to infer that the distance between them is (very nearly) $x + y$, and hence (approximately*) equal to the length, L, of the equivalent simple pendulum.

* The degree of approximation to the true value may be inferred in a given case, from the sensitiveness of the instrument, as concerns reversibility, to a slight change in the distance between the knife-edges.

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RATE OF CLOCK.

[Not required in 1884-85.]

Rating Time-pieces by Methods of Coincidence.

The seconds-clock, and a chronometer, each with two-seconds break-circuit, are connected in multiple arc with a battery and sounder. The chronometer is provided with a compensating balance, and is carefully rated by the standard time from the Observatory. In general, one or both circuits are flowing through the sounder, which consequently remains silent; but when both the clock and the chronometer break at the same instant of time, the armature is released, and strikes several blows at intervals of two seconds. The time of the last audible stroke is noted as before for several successive periods of coincidence, and thus the mean time is found during which the clock gains or loses two seconds with respect to the chronometer. If the latter beats sidereal, and the former mean solar time, these periods will be about 12 minutes and 10.5 seconds apart, the exact ratio being 1.00273796. In any case, the rate of the clock is easily calculated when that of the chronometer is given.

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The temperature, which has an important influence upon the rate of the seconds-clock, should be carefully observed at different times during the experiment, in order to be able to compare results obtained on different days.

The time of a pendulum is lengthened by an elevation of temperature of a single degree, centigrade, in the ratio 0.000006 for a steel shaft and .000010 for brass, corresponding respectively to a loss of half, and to a loss of four-fifths of a second in a day.

A third time-piece—the minute-clock—is now compared with the seconds-clock, or with an electric sounder connected with it, so that the beats of both are distinctly audible, and the periods of coincidence are determined by the ear alone. The times are read from the face of the time-piece, and the reductions are made in the same way as before.

The watch used in earlier determinations of times of coincidence may now be compared to advantage before and after the experiment with the standard clock. Unless the rate is unusually at fault, no correction need be made in the results previously obtained.

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THE CHRONOGRAPH.

[Not required in 1884-85.]

The Determination of the Frequency of a Tuning-fork by means of the Chronograph.

A slow tuning-fork, armed with a style (and lens), is set in vibration by a well-resined bow, and made to draw a wave-line upon the revolving drum of a chronograph, while, directly underneath, the time is marked by another style, borne by an armature in the seconds break-circuit. Care must be taken to have the drum evenly smoked, and to raise or lower it after or during each revolution, in order that the record may not become obliterated. The waves are then numbered, consecutively, with a sharp point, and the position of each break is determined in wave-lengths and fractions of a wave-length. Combining the first third and last third of the series, as in preceding experiments, we find the average number of waves executed in one second, that is, the *frequency* of the fork.

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LISSAJOUS' CURVES.

[Not required in 1884-85.]

The Determination of Musical Intervals by means of Lissajous' Curves.

The tuning-fork whose frequency has been determined is set in vertical vibration, and a second fork, sounding some harmonic of the first, is made to vibrate horizontally. A bright spot or bead upon the latter is viewed through the lens carried by the former, and thus partakes virtually of two motions at right-angles. While any simple ratio between the periods will cause the spot to trace out some figure, harmonics may generally be recognized by the appearance of lobes as in the figure 8. The number of lobes is approximately the ratio of the frequency of the forks; if, however, the musical interval be not quite exact, the figure will seem to revolve, assuming alternately a serpentine and a lobed appearance. Determining by the watch the frequency of the successive appearances of a right-handed figure, or that of a left-handed figure, we have the difference of the frequency of the higher fork from the nearest harmonic of the lower. The musical interval is then adjusted by weighting one of the forks until the figures cease to revolve. Since the weight reduces the frequency of the fork to which it is applied, the sign of the correction is determined.

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L VELOCITY OF SOUND.

[Omitted in 1884-85.]

The Determination of the Frequency of an Echo and the Velocity of Sound by means of a Metronome.

A position is chosen near one of the parallel walls of two buildings, separated by a measured distance of about a hundred metres, and a metronome is adjusted so as to coincide both with the report and echo of a sharp sound. The echo should be audibly repeated several times, and the mean reading of the metronome must be corrected by comparing it with the seconds clock after the experiment.

The product of twice the distance between the walls of the buildings by the frequency of the successive echoes gives the velocity of sound in air.

The result obtained in this way is nearly independent of the direction and velocity of the wind, and is to be compared with the value tabulated for the observed conditions of temperature and humidity.

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Example.

Distance between south wall of Jefferson Physical Laboratory and north wall of Lawrence Hall .	80.0 metres.
Frequency of metronome coinciding with sound and echo produced by concussion, mean of 10 observations	125 per minute.
Corrected frequency by clock . . .	2.1 per sec.
Velocity of sound, $2 \times 80 \times 2.1 =$.	33.6 $\frac{\text{metres}}{\text{sec.}}$
Velocity tabulated for 10° and 60% relative humidity, ...	33,856 $\frac{\text{cm.}}{\text{sec.}}$

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RESONANCE TUBE.

[Not required in 1884-85.]

The Determination of the Velocity of Sound in a Resonance Tube.

Certain lengths of a column of air respond to a tuning-fork. One of these is approximately equal to a quarter-wave, the others are longer than the above by *exactly* 1, 2, 3, etc., half wave-lengths; hence by finding two successive lengths of a variable tube at which resonance occurs, and taking the difference, the length of the half wave is accurately determined, hence that of the wave which, multiplied by the number emitted in one second, gives evidently the velocity of sound.

The tuning-fork whose rate has been determined is set in vibration and held over the open end of a glass jar of sufficient depth,* into which water, covered with oil to prevent evaporation, is admitted through a glass tube connecting with the bottom, until the air above, refreshed by the foot-blast, responds to the note emitted by the fork. Slight changes in the quantity of liquid, brought about by the contraction of a small elastic bulb, will cause more or less variation in the loudness of

* See Trowbridge, Experiment 162.—The depth must be at least three times that recommended by Thresh, page 142, (*b*); that is, for a fork of about 250 vibrations, it must exceed one metre. For slower forks, the depth must be proportionately greater.

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the tone, and the maximum occurs for some level easily determined between two limits, observed in a number of cases, the mean of which closely represents its true position.

The tube is now slowly filled with water, and although other maxima will be found, the principal resonance occurs only at two or more levels half a wave-length apart, and dividing the tube into segments nearly proportional to the odd numbers 1, 3, etc. Owing, however, to the spreading of the vibration beyond the end of the tube, the first segment is shorter than a quarter-wave by about one-fourth of the diameter of the orifice. The actual difference should be calculated for its intrinsic value, although the present method is independent of this source of error.

The several positions of the water-level at which resonance occurs are marked in all cases with a fine line, the distances being measured by a graduated rod or tape, and the value of a wave-length is found. Multiplying the latter by the frequency of the fork, we have the velocity of sound.

This velocity increases by very nearly 60 centimetres per second for every degree centigrade, and is also effected by hygrometric conditions;

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therefore the temperature and relative humidity of the atmosphere must obviously be taken, so that the result may be compared with the table below.

TABLE FOR THE VELOCITY OF SOUND IN AIR, IN CM. PER SEC.

Relative Humidity at 76 cm.	0%	20%	40%	60%	80%	100%
0°	33,220	33,225	33,231	33,236	33,242	33,247
1°	33,281	33,286	33,292	33,298	33,304	33,310
2°	33,341	33,347	33,353	33,360	33,367	33,373
3°	33,402	33,408	33,415	33,422	33,429	33,436
4°	33,462	33,469	33,476	33,484	33,491	33,499
5°	33,523	33,530	33,538	33,546	33,554	33,562
6°	33,583	33,591	33,600	33,608	33,617	33,625
7°	33,643	33,652	33,661	33,670	33,679	33,689
8°	33,703	33,713	33,722	33,732	33,742	33,752
9°	33,763	33,773	33,784	33,794	33,805	33,815
10°	33,823	33,834	33,845	33,856	33,867	33,879
11°	33,882	33,894	33,906	33,918	33,930	33,942
12°	33,942	33,955	33,967	33,980	33,993	34,006
13°	34,001	34,015	34,029	34,043	34,056	34,070
14°	34,060	34,075	34,090	34,105	34,119	34,134
15°	34,120	34,136	34,151	34,167	34,183	34,198
16°	34,179	34,196	34,213	34,229	34,246	34,263
17°	34,238	34,256	34,274	34,292	34,310	34,328
18°	34,297	34,316	34,335	34,354	34,374	34,393
19°	34,356	34,376	34,397	34,417	34,438	34,458
20°	34,415	34,436	34,458	34,480	34,502	34,524
21°	34,474	34,496	34,520	34,543	34,566	34,589
22°	34,532	34,557	34,581	34,606	34,630	34,655
23°	34,590	34,617	34,643	34,669	34,695	34,722
24°	34,649	34,677	34,705	34,732	34,761	34,789
25°	34,707	34,737	34,766	34,796	34,826	34,856
26°	34,765	34,797	34,828	34,860	34,892	34,924
27°	34,823	34,857	34,890	34,924	34,958	34,992
28°	34,881	34,917	34,953	34,988	35,025	35,061
29°	34,939	34,977	35,015	35,053	35,092	35,130
30°	34,997	35,037	35,077	35,118	35,158	35,199
31°	35,055	35,097	35,139	35,182	35,225	35,269
32°	35,113	35,157	35,202	35,247	35,293	35,340
33°	35,170	35,218	35,265	35,313	35,362	35,412

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EXAMPLE.

Resonance reaches maximum during contraction of bulb, mean of 9 observations, between	111.6 cm.
and	110.8
Position of maximum*	111.2 cm.
Resonance reaches maximum, mean of 7 observations, between	35.5 cm.
and	34.7
Position of maximum	35.1 cm.
Half wave-length, (111.2—35.1)	76.1 cm.
Quarter wave-length	38.1
Length of first segment	35.1
Correction for open end	3.0
Compare with quarter diameter of tube	2.6
Length of whole wave	152.2 cm.
Frequency	227.5
Velocity of sound, observed	34,626 $\frac{\text{cm.}}{\text{sec.}}$
Velocity tabulated for 19° and 80 per cent. relative humidity	34,438 $\frac{\text{cm.}}{\text{sec.}}$

* The distances are all measured from the open end of the tube.

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LONGITUDINAL VIBRATIONS.

[Not required in 1884–85.]

*The Determination of the Velocity of Sound and Modulus of Elasticity by the Method of Longitudinal Vibrations.**

First, a column of air in a tube of measured length, open at both ends, is set in vibration by the foot-blast, and the pitch being found by means of a graduated pipe, the frequency is approximately given by the table below.

Second, a metallic wire of measured length, being set in longitudinal vibration by a piece of resined cloth, is stretched between two vises until the note no longer rises, and the frequency is determined in the same way.

* A good musical ear is absolutely requisite for this experiment.

TABLE FOR AMERICAN CONCERT PITCH.

A.....	57.13	114.3	228.5[2]	457.0[5]	914
A sharp	60.52	121.0	242.1	484.2	968
B.....	64.12	128.2	256.5	513.0	1026
C.....	67.93	135.9	271.7[3]	543.5	1087
C sharp	71.97	143.9	287.9	575.8	1152
D.....	76.26[1]	152.5	305.0[4]	610.0	1220
D sharp	80.79	161.2	323.2	646.3	1293
E.....	85.58	171.2	342.4	684.7	1369
F.....	90.68	181.4	362.7	725.4	1451
F sharp	96.07	192.1	384.3	768.6	1537
G.....	101.79	203.6	407.1	814.3[6]	1629
G sharp	107.84	215.7	431.4	862.7	1725

[1]Lowest D of Bass Voice.
[2]Assumed.
[3]Middle C of Piano.
[4]Lowest D of Flute.
[5]Violin A.
[6]Highest G of Treble Voice.

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For greater exactness, however, the following method may be adopted. The pitch-pipe and siren are both connected with a gasometer filled with air by the foot-blast, and the weights being so adjusted that the two notes are nearly in unison, as shown by the slowness of the beats, the pressure of the hand will be sufficient to compensate for any variation which is likely to occur. The dial, previously set at zero, is connected with the running gear at the stroke of the minute-clock, and disconnected after several strokes, making, if possible, in the mean time, a reading of the dial at every stroke of the minute clock.

Rapid beats must not be allowed to occur, but the error due to slow beats* may be almost neutralized by having the pressure now too little and now too great.

Finding, by the method peculiar to consecutive observations, the average number of vibrations registered per second, we have the frequency of the note given out by the pitch-pipe, which should agree with that emitted by the tube or wire both before and after the experiment, care

*The number of beats may actually be counted by an assistant if the pressure is sufficiently constant, and added or subtracted according to circumstances from the number of vibrations registered.

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being taken to sound the pitch-pipe always in the same way.

The product of the frequency by twice the length plus the diameter of the vibrating column* is equal in either case to the velocity of sound, v; from which, knowing the density,† D, the modulus of elasticity, E, is calculated by the formula

$$E = v^2 D.$$

It is important in the case of air to determine the ratio of this modulus of elasticity to the pressure in dynes, which is found by multiplying together the reading of the barometer in centimetres, the weight of a cubic centimetre of mercury in grammes, (13.596 at 0°) and the value of the gramme in dynes, that is, the acceleration of gravity, already determined.‡

The modulus of elasticity for solids, determined in this way, is said to be 1 or 2 per cent. higher than that obtained by stretching.§

* The open tube of air vibrates like a rod clamped in the middle, which has the same rate as one set at both ends.

† For the density of air, see the tables under Capacity of Vessel; for steel $D = 7.8$; for brass, 8.5, nearly.

‡ The ratio should be 1.41, nearly, and will be of use, later on, in determining the relation between the specific heat of air and its mechanical equivalent.

§ See Kohlrausch, page 86, lines 3—8, also Everett's Units and Physical Constants, § 144, pages 116—7.

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EXAMPLE.

Length of tube	49 cm.
Twice ditto	98
Diameter	2
Wave length	100 cm.
Frequency of $E + \frac{1}{2}$ semitone, from table	352.5
Frequency by siren	345.0
Velocity of sound ($345. \times 100$)	$3.450 \times 10^4 \frac{\text{cm.}}{\text{sec.}}$
Density of air at 19°, 76 cm., and dew-point 18°	.0012
Elasticity ($(34{,}500)^2 \times .0012$)	1.4283×10^6
Pressure in dynes ($76 \times 13.596 \times 981$)	1.0137×10^6
Quotient	1.409

Length of steel wire	520.0 cm.
Length of wave	1040.0 cm.
Frequency of note, B, from table	513.0
Frequency by siren	506.0
Velocity of sound $506 \times 1040 =$	$5.262 \times 10^5 \frac{\text{cm.}}{\text{sec.}}$
Density by hydrostatic balance	7.849
Elasticity ($(526{,}200)^2 \times 7.849$)	2.173×10^{12}
Young's modulus, acc. to Everett,	2.139×10^{12}

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NOTE. The conditions which are assumed in establishing the formula for the velocity of sound are 1st, that the actual velocity, w, of a particle is small as compared with the velocity of the wave; 2d, that the changes of density are small as compared with the normal density; and 3d, that there is no transfer of matter as a whole either in the direction of the wave or in the opposite direction.

It is evident that similar portions of the two waves must be propagated with the same velocity, v, the conditions being the same; and that dissimilar portions can not travel with different velocities, since if this were so the wave would be obliterated.

The weight of air traversed by an imaginary unit cross-section moving with the wave so as to remain always in the same portion whether dense or rare, must in all cases be the same, since otherwise between two such cross-sections there would be an indefinite accumulation or exhaustion of matter, contrary to the second condition.

But the weight traversed is the product of the density, D, by the relative velocity $(v - w)$, so that at two points, 1 and 2, designated by subscripts,

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$(v - w_1)\ D_1 = (v - w_2)\ D_2$, whence we have $\frac{D_1 - D_2}{D_2} = \frac{w_1 - w_2}{v - w_1} = \frac{D_1 - D_2}{D}$ (nearly) $= \frac{w_1 - w_2}{v}$ (nearly),

by the first and second conditions. But by the definition of the modulus of electricity, E, we have

$\frac{D_1 - D_2}{D} = \frac{P_1 - P_2}{E}$, whence substituting,

$P_1 - P_2 = \frac{E(w_1 - w_2)}{v}$, where P_1 and P_2 denote the pressure per unit of surface at the two points in question. If now the points are contained in the two opposite faces of a unit cube perpendicular to the line of propagation, $P_1 - P_2$ is the force acting upon this cube, whose mass is evidently equal to the density, D; and we see that while the wave is passing from the point 1 to the point 2, which being the unit of distance, occupies the time, $\frac{1}{v}$, the point 2 assumes the characteristics previously belonging to the point 1, that is, the velocity changes from w_2 to w_1 in the time $\frac{1}{v}$, and equating the impulse and the momentum, we have,

$P_1 - P_2 \times \frac{1}{v} = E\ \frac{w_1 - w_2}{v} \times \frac{1}{v} =$

$D \times (w_1 - w_2)$; whence $E = v^2 D$.

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ELECTRIC MICROMETER.

The Determination of the Modulus of Elasticity by means of an Electric Micrometer.

By observing the elongation produced by a given weight in a wire of measured length and thickness, Young's modulus of elasticity is determined.

Multiplying the weight by the acceleration of gravity, already found, we have the force, F, in dynes which divided by the area of cross-section $\mathcal{Q}$ (equal to πr^2 if r be the semi-diameter) gives the absolute measure of the force per square centimetre, that is, the *stress* exerted upon the wire.

Dividing the total stretching, S, by the length of the wire, L, we have the lengthening of each centimetre, that is, the *strain* produced; and the quotient of the stress by the strain is the modulus of elasticity in question, given accordingly by the fomula,

$$E = \frac{FL}{\mathcal{Q}S}.$$

The total length, L, subject to stretching, is measured by a graduated steel tape, and the diameter, found by a common micrometer gauge, is carefully corrected for the zero error.

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The electric micrometer, supported upon a solid pier, is then connected with a battery and sounder so as to complete the circuit when its point touches a flat piece of platinum soldered to a ring sufficiently heavy to straighten the wire. The screw is so adjusted that the least vibration causes a commotion of the sounder.

The necessary* weight, tested accurately enough by ordinary scales, is then removed from a platform suspended from the same support as the wire, and placed in the pan hanging from the ring already alluded to. The screw must be adjusted once more, and the difference in the two readings of the graduated head† gives the amount which the wire has been stretched. When the weight is replaced on the platform, the ring should return to its former position. Care must be taken to withdraw the screw before altering the weight, since otherwise the point would be injured, and the place of contact upon the platinum should not be disturbed by changing the load.

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* The weight should be about half that necessary to break the wire, but not sufficient to produce any permanent elongation.

† The correctness of the pitch of the screw may be tested by measuring the thickness of a metallic plate, the method being obvious, and comparing the result with the indications of the vernier and micrometer gauges.

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The yielding of the point of suspension may be disregarded in this experiment, since the weight depending upon it is unchanged; but the support should be as firm as possible, and the measurements must be repeated until a close agreement is obtained.

Example.

Length of iron wire, ... 684 cm.
Diameter, mean of 100 obs. . . . 0.0660 cm.
Semi-diameter . . . . . 0.0330 cm.
Cross-section, $3.1416 \times .033^2 = 0.00342$ sq. cm.
Load . . . . . 5.00 kilos.
Force, $981 \times 5 \times 1000 = 4.905 \times 10^6$ dynes.
Mean reading of electric micrometer
with load, 10 obs. . . . . 1.223 cm.
without load, 10 obs. . . . . 0.703 cm.
Stretching . . . . . 0.520 cm.

Young's Modulus, $\frac{4.905 \times 10^6 \times 684}{.00342 \times .52} = 1.89 \times 10^{12}$

Values tabulated for iron . $\begin{cases} \text{from } 1.745 \times 10^{12} \\ \text{to } 1.963 \times 10^{12} \end{cases}$

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YOUNG'S MODULUS.

[Not required in 1884-85.]

The Determination of Young's Modulus of Elasticity by means of the Cathetometer.

A portion of a uniform straight wire of length, L, and diameter, D, subjected to the force, F, undergoes the stretching, S. Young's modulus of elasticity is then given by the formula,*

$$E = \frac{4FL}{\pi D^2 S}.$$

Except that a position and not the whole wire is taken into consideration, the experiment differs from that with the electric micrometer only in the method of determining the stretching, S, and in a peculiar device by which the yielding of the support may be completely eliminated.

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* For the proof of this formula, see under the Electric Micrometer.

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The diameter of the wire must be measured with the utmost precision by the micrometer gauge, or better with the hydrostatic balance.*

The wire is then suspended from as firm a support as possible, and a scale-pan is attached with load enough to straighten the wire. Two parallel wires are joined on at or near the point of suspension, and carry just above the scale-pan a heavy cross-bar perforated for the central wire. The distance is carefully measured along the latter between the junction and a fine scratch just above the cross-bar, and two other scratches are made at about the same height upon the parallel wires. By means of the vernier of the cathetometer,† the position of each is repeatedly determined with and without the addition of a given weight, so that any permanent strain may be detected.

The mean difference of the readings, after deducting that of the parallel wires due to the yielding of the support, gives evidently the stretching of the portion of the wire which has been measured, from which, finally, we calculate the modulus of elasticity.

* The cross-section may be found directly by this method when practicable, being the quotient of the displacement by the total length of wire.

† The cathetometer must be properly levelled and the telescope must not be disturbed in any way after the series of measurements is once begun.

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EXAMPLE.

Length of steel wire . . . 660.0 cm.
Diameter (mean of 100 observations) 0.02224 cm.
Cross-section ($3.1416 \times .01112^2$) 0.0003885 sq.cm.
Weight added . . . . 2,000. g.
Acceleration of gravity, observed . $981 \frac{\text{cm.}}{\text{sec. sec.}}$
Absolute weight of 2,000 g. 1.962×10^6 dynes

Readings.

	Left hand wire.		Right hand wire.		Central wire.	
	Not weighted	Weighted	Not weighted	Weighted	Not weighted	Weighted
	71.052		71.048		72.567	
		71.041		71.037		71,001
	71.051		71.047		72,566	
		71.040		71.037		71.000
	71.050		71.046		72.564	
		71.038		71.035		70.998
	71.049		71.045		72.565	
Mean	71.0505	71.0397	71.0465	71.0363	72.5655	70.9997

Deflection of left hand wire . 0.0108 cm.
" " right " " . . 0.0102 cm.

Mean . . . . . 0.0105 cm.
Deflection of central wire . . 1.5658 cm.

Stretching " " " . . 1.5553 cm.
Young's modulus, $\frac{1{,}962{,}000 \times 660}{.0003885 \times 1.5553} = 2.143 \times 10^{12} \frac{\text{dynes.}}{\text{sq. cm.}}$

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DEFLECTION OF BEAM.

The Determination of the Modulus of Elasticity by Transverse Flexure.

A beam of measured length, L, breadth, B, and thickness, T, is supported at both ends, and loaded in the middle with a given weight, equal to F in absolute measure. Observing the deflection, D, the modulus of elasticity is given by the formula*

$$E = \frac{FL^3}{4DBT^3}$$

The formula is established as follows. The tangents or normals at any two points separated by the unit of distance are inclined by an amount which is the measure of the curvature, c, and determines the compression and extension, respectively, of the upper and lower portions of the beam. The maximum strain is seen in either case to be the product of the curvature and the half-thickness, and the average strain, s, is equal to half the maximum, or $s = \pm \frac{1}{4} cT$. The average stress, p, is the product of s by the modulus of elasticity, that is, $p = \pm \frac{1}{4} cET$.

* Proof of formula not required in 1884-85.

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The two areas subject to this stress in opposite directions are equal each to $\frac{1}{2} BT$, so that the total force, $f = \pm \frac{1}{8} cEBT^2$. The point of application of any system of parallel forces regularly increasing from a value zero to a maximum is twice as near the position of the latter as to that of the former, so that the arm of the couple produced is in this case two-thirds the thickness of the beam; therefore the moment,

$$m = f \times \tfrac{2}{3} T = \tfrac{1}{12} cEBT^3.$$

But if F is the absolute value of the deflecting force at the centre, half of which is felt at the nearer extremity of the beam, whose distance from the unit section considered is l, we must have $m = \frac{1}{2} Fl$, whence $c = \dfrac{6Fl}{EBT^3}$.

Regarding the centre as fixed and the ends as movable, we see that on either side, the vertical yielding, y, due to the curvature, c, of the unit of length is evidently cl, that is $y = \dfrac{6Fl^2}{EBT^3}$, showing that the yielding due to different unit sections of the beam increases as the square of their distance from the extremity, and hence that the average value, like that of the cross-section of a

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pyramid, which follows the same law, will be equal to one-third of the maximum value, Y, due to the unit of length at the centre, for which

$$Y = \frac{6F\left(\frac{L}{2}\right)^2}{EBT^3},$$

so that the total deflection due to $\frac{L}{2}$ units of length, will be,

$$D = \frac{1}{3} Y \times \frac{L}{2} = \frac{2F\left(\frac{L}{2}\right)^3}{EBT^3}, \text{ whence}$$

$$E = \frac{1}{4} \frac{FL^3}{DBT^3}, \text{ Q. E. D.}$$

The beam should have a rectangular section, measured in several parts by the micrometer gauge. It must be firmly supported upon two knife-edges, the distance between which is determined by the metre rod. The weight is attached nearly at the middle, and the deflection is found by the difference of the readings of an electric micrometer, taken repeatedly with and without the weight. The beam should return to its former shape upon removal of the weight, and care must be taken not to injure the platinum point

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of the screw. The upper surface of the beam must be brightly polished in the neighborhood of the point of contact, and the connections with the battery and sounder should be carefully made, so that the latter may indicate the lightest touch.

Each whole turn of the screw may be assumed to represent a millimetre; the revolutions are counted and divided into hundredths and thousandths, just as in the case of the micrometer gauge.

EXAMPLE.

Distance between knife-edges . . 100.00 cm.
Breadth of beam
(mean of 25 observations) . . 0.7000 cm.
Thickness of beam
(mean of 75 observations) . . 0.7095 cm.
Weight added . . . . . 2,000 g.
Acceleration of gravity, observed . 981 $\frac{\text{cm.}}{\text{sec. sec.}}$
Absolute value of weight . . 1.962×10^6 dynes.
Deflection (mean of 5 observations) 0.9125 cm.
Modulus of elasticity:

$$\frac{1{,}962{,}000 \times 100^3}{4 \times .9125 \times 0.7 \times 0.7095^3} = \ldots \quad 2.15 \times 10^{12} \frac{\text{dynes}}{\text{sq. cm.}}$$

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TORSION PENDULUM.

The Determination of Simple Rigidity by means of the Torsion Pendulum.

A cylinder of mass, M, and radius, R, being suspended with its axis vertical by a fine wire of length, l, and radius, r, the time, t, of oscillation under torsion is determined; the simple rigidity is then given in absolute measure by the formula,

$$Z = \frac{\pi l M R^2}{r^4 t^2} .$$

The wire is suspended from an oblique terminal groove in a bracket, and is attached by a small clamp to the centre of a metallic disc, so that the free length is exactly equal to that, for instance, of the metre rod. The diameter of the wire must be measured with the utmost precision by the micrometer gauge, or determined by the hydrostatic balance; that of the disc is found with sufficient accuracy by a vernier reading to tenths of a millimetre, and the mass by a common balance. Care must be taken to prevent the wire from twisting beyond the supposed limits, and the disc must revolve in a horizontal plane.

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The time of oscillation is found by noting the times of ten or more successive turning-points, and reducing by the method used for the coincidences of the simple pendulum.

The rigidity calculated by the formula should be two or three times* less than the modulus of elasticity, since it takes two or three times less force to make two parallel surfaces slide by one-another, than to make them approach or recede by a given amount.

The formula† is established as follows:—suppose the cross-sections of the cylinder and wire to be divided similarly each into a given number of small parts, and let A and a be the areas, and X and x the distances from the axis, respectively, of any two parts similarly situated, one in the cylinder and the other in the wire. If now the wire be twisted through the unit of angle, so that the moment of the resistance brought into play is by definition equal to the directive force, D, the tangential strain upon the small surface considered will be $\frac{x}{l}$, and since the simple rigidity is the ratio of the tangential stress to the strain pro-

* The "Coefficient of Torsion," used by Kohlrausch and other writers, is equal to one-half of the Simple Rigidity.

† Proof of formula not required in 1884-85.

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duced, this stress is $\frac{xZ}{l}$, the force upon the area, a, is $\frac{axZ}{l}$, and its moment about the azis of the wire is $\frac{ax^2Z}{l}$.

The moment of inertia of the corresponding part of the cylinder covered by the area, A, is of course $AX^2H\Delta$, if H denotes the height and Δ the density of the cylinder. Hence the ratio of the moment of inertia, k, to the directive force, d, due respectively to any two homologous parts of the cylinder and wire, is

$$\frac{k}{d} = \frac{lAX^2H\Delta}{ax^2Z} ;$$

but by geometry, $A : a :: R^2 : r^2 :: X^2 : x^2$, hence

$$\frac{k}{d} = \frac{lHR^4\Delta}{r^4Z} ,$$

which being constant must also express the ratio between the total moment of inertia, K, and the total directive force, D. But from Kater's pendulum we have, substituting,

$$t^2 = \frac{\pi^2K}{D} = \frac{\pi^2lHR^4\Delta}{r^4Z} ,$$

whence, remembering that $M = \pi HR^2\Delta$, we find

$$Z = \frac{\pi lMR^2}{r^4t^2} , \text{ Q.E.D.}$$

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Example.

Length of steel wire	100.0 cm.
Diameter " " average . .	0.02200 cm.
Radius " " " . .	0.01100 cm.
Diameter of disc, average	10.00 cm.
Radius " " "	5.00 cm.
Time of oscillation	25 sec.

Simple Rigidity,

$$\frac{3.1416 \times 100 \times 1000 \times 5^2}{.0110^4 \times 25^2} = 8.58 \times 10^{11}$$

Value according to Everett . . . 8.19×10^{11}

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STRENGTH AND FRICTION.

The Determination of Breaking Strength, Maximum Extensibility, and Coefficient of Friction.

I. A short piece of wire is broken by a spring balance, and the necessary force, expressed in dynes, is divided by the cross-section, determined as previously by the micrometer gauge, to find the breaking strength of a column whose section is a square centimetre.

Care must be taken to have no flaw or sudden bend in the wire, which should part somewhere in the straightened portion. The maximum reading of the spring balance is carefully observed in a number of cases, the average of which is taken, and corrected for the zero reading under the same conditions. By means of suitable weights, the graduation is tested, and the correction, if any, having been applied, the result is reduced to absolute measure as in previous experiments.

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The ratio of the breaking strength to the modulus of elasticity gives evidently the greatest strain which the substance can bear, that is, the *maximum extensibility* of the unit of length of the given material.

II. Two smooth, flat surfaces of the same substance as the wire are inclined until the upper, carrying a moderate load, slides, when once started, with constant velocity over the lower. The angle of inclination is measured in a number of cases by Abney's Level, and the mean is taken, the tangent of which is equal to the coefficient of friction.

EXAMPLE.

I. Steel wire, cross section .0003885 sq. cm.

No. of trial.	Place of Breaking.	Maximum reading of balance.
1	Pieces lost	[8300] g.
2	Fault near hook	[6500]
3	Middle	8900
4	Near hook	8400
5	Middle	9200
6	"	8700
7	Fault near hook	[7300]
8	Slipped	[7500]
9	Near bend	9200
10	Knot beyond bend	[8000]
11	Middle	8700
12	Close to hook	9600
13	" " "	9000
14	Middle	8600

Mean, rejecting Nos. 1, 2, 7, 8 and 10 . . 8910 g.

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Zero reading of balance (horizontal) —210 g.
Ditto, vertical, with wire . . . —50 g.
Reading with 9000 g. . . . 9000 g.

Deflection for 9000 g. . . . 9050
Correction for graduation . . —50 g.
True breaking strength
$(8910 + 210 - 50) =$ 9070 g.
Ditto in absolute measure
$(981 \times 9070) = 8.91 \times 10^6$ dynes
Breaking strength per sq. cm. 2.29×10^{10} dynes
Young's modulus, by experiment, 2.143×10^{10}
Maximum extensibility
$2.29 \times 10^{10} \div 2.143 \times 10^{12} = 0.0107$

II. Two surfaces of sheet iron; load 100 g.

Angle of inclination producing slow continuous motion.

1.	16° 47′	6.	16° 30′
2.	17° 7′	7.	16° 8′
3.	17° 8′	8.	16° 8′
4.	16° 12′	9.	15° 50′
5.	15° 32′	10.	16° 50′

Mean angle of inclination . . 16° 42′
Coefficient of friction (tangent 16° 42′) = 0.300

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CAPILLARY TUBE.

The Determination of Surface Tension and Viscosity by Means of a Capillary Tube.

I. A piece of graduated thermometer-tubing of length, l, is filled with mercury, and the mass, m, is found by depositing the contents ten or more times into the pan of a delicate balance, properly protected against amalgamation. If δ be the density of the mercury (13.55 at 20°), the radius of the tube is given by the formula,

$$r = \sqrt{\frac{m}{\pi\delta l}} = \sqrt{\frac{0.0235\ m}{l}}\ , \text{ nearly, at } 20°.$$

The tube carefully cleaned, if necessary, with nitric acid, is filled with water and supported vertically with its foot in the liquid, and from the mean of several observations the height, h, in centimetres is found, at which the water stands in the tube above the free surface in the vessel.

If the surface tension be s, the contractile force of the film will be $2\pi rs$, which must balance the weight of the column, $\pi r^2 ghd$, where d, the density of water, is very nearly equal to unity. Hence we have

$$s = \tfrac{1}{2}\, ghdr = 490\ hr, \text{ nearly.}$$

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II. The tube is then connected with a Mariotte's Bottle, whose inlet is at a measured height, *H*, in centimetres above the point of delivery, and the quantity of water *w*, in grammes, passing through the tube in a given length of time, *t*, is determined. If the water be properly filtered, the drops will be formed at regular intervals.

The tube is wiped perfectly dry by loose cotton, which will absorb the flow until, at the stroke of the minute clock, it is suddenly removed and inserted into the narrow neck of a small bottle, suited to the purpose. Care must be taken to avoid evaporation by covering the surface of the water with a film of oil; and all danger from leakage at the junction of the capillary and rubber tubes is avoided by winding them tightly with hemp and surrounding with loose cotton, a little of which, previously weighed, is also packed around the tube in the neck of the bottle.

After several minutes, the tube is wiped and allowed to drain into this cotton, and is then suddenly removed at the next stroke of the clock.

The bottle is immediately corked, and the total gain of weight gives the quantity of liquid which has flowed through the tube in the given

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time, and the coefficient of viscosity is calculated by the formula,

$$\eta = \frac{\pi g d^2}{8} \cdot \frac{Hr^4 t}{lw} = 384 \frac{Hr^4 t}{lw}, \text{ nearly, at } 20°.$$

The formula is established as follows. Let the tube be divided into a great number of tubular sections having a common axis and unit thickness, which latter may be disregarded in comparison with the radius, x. The force tending to produce a sliding of the inside of a given tube and its solid contents with respect to the outside is $Hgd \times \pi x^2$, disregarding half the thickness of the section. The resistance to this motion is, according to definition, the continued product of the coefficient of viscosity, η, the difference, v, between the internal and external velocities, and the area, $2\pi xl$, of either of the sliding surfaces; so that for steady flow we have

$$\eta v . 2\pi x l = Hgd . \pi x^2, \text{ or}$$

$$v = \frac{Hxgd}{2\eta l} .$$

If the tubular section whose radius is x, should be suddenly solidified, that is, if the inner and outer surfaces should become relatively fixed, the

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mutual sliding of other surfaces remaining the same, the velocity of all outside points, being determined by the outermost layer, which is always at rest, would be unchanged, while that of all points in the interior would be diminished by the amount, v; hence the flow would be less by the quantity,

$$q = \pi x^2 . v = \frac{\pi H x^3 g d}{2\eta l} ,$$

which measures the portion of the flow due to the fluidity of the tubular section whose radius is x.

The average value of q, like the moment of the cross-section of a pyramid about the apex, depending on the cube of the distance, x, is one-fourth as much as if x were continually equal to r, its maximum value; and as there are r tubular sections, we have for the total flow in cubic centimetres per second,

$$\mathfrak{Q} = \frac{w}{td} = r \times \tfrac{1}{4} \left(\frac{\pi H r^3 g d}{2\eta l} \right) = \frac{\pi H g r^4 d}{8\eta l} ,$$

whence $\eta = \frac{\pi g d^2}{8} \cdot \frac{H r^4 t}{w l}$, Q. E. D.

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EXAMPLE.

I. Length of tube . . . . 19.70 cm.
Weight of mercury contained . 0.0891 g.
Radius of tube, $\sqrt{\frac{0.0235 \times 0.0891}{19.70}} =$. 0.01031 cm.
Height of capillary column, (10 obs.) . . . . 14.70 cm.
Surface tension at 20°,
$\frac{1}{2} \times 14.7 \times 0.01031 \times 981 =$ 74.3 $\frac{\text{dynes}}{\text{cm.}}$
Compare Ganot, § 133, reduced 73. "
" page 42, Everett's Units, 20° 81. "

II. Head of water . . . . 700 cm.
Time of flow . . . . 480 sec.
Weight delivered . . . . 7.153 g.
Viscosity at 20°,

$$\frac{384 \times 0.01031^4 \times 700 \times 480}{19.70 \times 7.153} = 0.0103.$$

Compare with results of Meyer,* 15.5° 0.0131
" " " " Helmholtz† at 24.5° . . . 0.014061
Ditto at 100° . . . . 0.003?

* Daniell's Physics, page 274. † Lamb's Hydrodynamics, § 178.

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HYDRAULIC FRICTION.

The Determination of the Resistance of a Large Tube to the Flow of Water, and the Hydraulic Coefficient of Friction.

A rubber tube of measured length, l, is used as a siphon to draw water from a tank whose constant level is at a measured height, h, above the point of delivery. The orifice is fixed in position by a burette holder, and when the stream has attained its maximum velocity, an amount of water, m, is collected by a suitable vessel in a given time, t, and weighed by a rough balance. The capacity of the tube, c, is also determined in grammes of water of density, d, and the radius, r, is calculated by the usual formula,

$$r = \sqrt{\frac{c}{\pi l d}}.$$

The quotient of the mass, m, by the capacity, c, is evidently the number of tubefuls which have passed through the orifice, so that, multiplying by the length of the tube and dividing by the time, we have the velocity of efflux, v. The loss of pressure due to the generation of

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this velocity is equal to half its square; the remainder is the effective pressure, which multiplied by the area of the cross section, πr^2, is equal and opposite to the total resistance, R, given, accordingly, by the formula,

$$R = \pi r^2 \left(hgd - \tfrac{1}{2} v^2\right).$$

In large tubes, the resistance increases not as in capillaries, simply as the velocity, but as its square;* and is also proportional to the area of wetted surface, $2\pi rl$, so that the coefficient of friction in hydraulics may be defined by the formula

$$f = \frac{R}{2\pi r l v^2},$$

assuming that this coefficient is equal to the resistance of the unit of wetted surface to a flow of water with the unit of velocity.

The result is to be compared with the table below showing the different values obtained by Weisbach for different velocities in metallic tubes of medium size.

*The law of its variation is still in dispute. See Weisbach, *Theoretical Mechanics*, § 427 and § 428.

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EXAMPLE.

Length of rubber tube . . .	360 cm.
Capacity " " " . . .	240 g.
Radius " " " $\sqrt{\frac{240}{3.1416 \times 360 \times 1}}$.	0.461 cm.
Weight of water delivered . .	9,000 g.
Time occupied	180 sec.
Velocity of efflux $(\frac{9,000 \times 360}{240 \times 180})$ . .	75 $\frac{\text{cm.}}{\text{sec}}$
Loss of pressure $(\frac{1}{2} \times 75^2)$.	2.8×10^3 $\frac{\text{dynes}}{\text{sq. cm.}}$
Difference of level	50 cm.
Hydrostatic pressure (50×981)	49.1×10^3 $\frac{\text{dynes}}{\text{sq. cm.}}$
Effective " $(49.1 - 2.8)$	46.3×10^3 "
Total resistance $(3.1416 \times .461^2 \times 46,300)$	30.9×10^3 dynes
Coefficient of friction $(\frac{30,900}{2 \times 3.1416 \times .461 \times 360 \times 75^2})$	.00527
Value tabulated for $v = 75$ . .	.00316

TABLE FOR THE COEFFICIENT OF FRICTION OF WATER. (f.)

Reduced from Weisbach's Table. Velocity, v, in cm. per sec.

v	f	v	f	v	f	v	f	v	f
0	∞	100	.00299	200	.00264	300	.00249	400	.00239
10	.00554	110	.00293	210	.00262	310	.00248	410	.00239
20	.00445	120	.00288	220	.00260	320	.00247	420	.00238
30	.00396	130	.00284	230	.00258	330	.00246	430	.00238
40	.00368	140	.00280	240	.00256	340	.00245	440	.00236
50	.00347	150	.00276	250	.00255	350	.00244	450	.00236
60	.00333	160	.00274	260	.00254	360	.00243	460	.00235
70	.00321	170	.00271	270	.00253	370	.00242	470	.00235
80	.00312	180	.00269	280	.00251	380	.00241	480	.00234
90	.00305	190	.00266	290	.00250	390	.00240	490	.00234
100	.00299	200	.00264	300	.00249	400	.00239	500	.00233

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WATER-MOTOR.

[Not required in 1884-85.]

The Determination of the Maximum Efficiency and Rate of Working of a Water-motor.

The motor is driven by a measured pressure of water, p, in megadynes per square centimetre, and a mass of water, m, is delivered in a given time, t, during which the wheel of the motor, whose circumference is c, has made a number of revolutions, n, against the resistance of a friction-rim,* exerting the tangential force, f, in megadynes, upon a spring balance by which it is held in place.

The rate of working in megergs per second is then $\frac{cnf}{t}$, and the quotient of the latter by the rate at which energy is expended, namely $\frac{mp}{td}$, is the *efficiency*, that is the proportion of energy utilized, equal accordingly to $\frac{cnf}{mp}$, nearly, since d, the density of water, may be taken equal to unity.

Several trials must be made with different amounts of friction in order to compass the conditions of greatest efficiency and maximum rate of working. The latter will generally occur

* If instead of a friction-rim, a simple loop in a cord be employed, the indication of the spring balance must be diminished by the amount of the load necessary to generate the friction.

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when the motor is at about half speed; the former necessarily requires a somewhat less velocity.

The pressure of the water is measured by an ordinary gauge, graduated in pounds per square inch, and reduced to megadynes per square centimetre by multiplying by the factor 0.0690. The gauge may, if necessary, be tested with a mercury manometer, allowing 0.01328 megadynes per square centimetre for the vertical pressure of every centimetre of mercury at 20° and 0.00098 for every centimetre of water. The readings will be rendered more precise by slightly jarring the needle, so as to relieve the friction, and by nearly closing the stopcock to prevent rapid oscillations. The equivalent of the hydrostatic pressure of water between the gauge and the outlet of the motor should be added to the corrected reading.

Example.

Circumference of wheel, c, = . .	72 cm.
Correction for gauge, constant between 15 and 25 lbs. per sq. inch	$-0.04 \frac{megadynes}{sq.\ cm.}$.
Height of gauge above outlet .	40. cm.
Correction for ditto . .	$+0.04 \frac{megadynes}{sq.\ cm.}$.
Total correction, constant .	$0.00 \frac{megadynes}{sq.\ cm.}$.

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$$\frac{72 \times 73.0 \times 1.45}{12 \times 16.6} = \frac{6}{11} = .5$$

$r = \frac{n}{t}$; number of ...

s = ... $= pq = pn$

$v = 72r = cr$

$e = \frac{100n}{s}$

$f = .98(b-2)$

n = ... $= fv = fcn$

$q = 1000$ ...

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Observed:—

l. Load in kilos.	*b.* Mean reading of spring Balance in kilos.	*h.* Mean Head of water in lbs. per sq. in.	*n.* Number of revolutions of wheel, counted.	*t.* Duration of experiment in seconds.	*m.* Mass of water delivered in kilos.
.00	0.00	18.1	114.0	30	11.80
.00	0.31	18.3	100.0	30	10.85
.00	0.41	18.7	92.7	30	10.38
.00	0.62	19.2	80.7	30	9.57
.01	0.95	20.3	64.3	30	8.12
.02	1.11	21.3	114.0	60	14.80
.03	1.48	24.2	73.0	60	12.02
.05	1.61	24.5	57.5	60	10.80
.10	1.94?	24.9	26.0	58	8.30
.20	2.24?	25.0	0.0	60	6.78

Calculated:—

r. Number of Revolutions per second $= n \div t$.	*v.* Velocity of rim of wheel in cm. per sec. $= 72\ r$.	*f.* Friction proper in megadynes $= .98\ (b - l.)$	*w.* Work Utilized by machine in megergs per sec. $= fv$.	*p.* Pressure of water in megadynes per sq. cm. $= .069\ h$.	*q.* Quantity of water delivered in cu. cm. per sec. $= 1000\ m \div t$.	*s.* Work Spent upon machine in megergs per sec. $= hq$.	*e.* Per cent. of Efficiency of machine $= 100\ w \div s$.
3.80	274	0.00	0.	1.25	393.	491.	0.0
3.33	240	0.30	72.	1.26	362.	456.	15.8
3.09	222	0.40	89.	1.29	346.	446.	20.0
2.69	194	0.61	118.	1.32	319.	421.	28.0
2.14	154	0.92	142.	1.40	271.	379.	37.5
1.90	137	1.07	147.	1.47	247.	363.	40.5
1.22	88	1.42	125.	1.67	200.	334.	37.4
0.96	69	1.53	106.	1.69	180.	304.	34.9
0.45	32	1.80?	58.	1.72	143.	246.	23.6
0.00	0	2.00?	0.	1.73	113.	195.	0.0

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JOULE'S EXPERIMENT.

[Omitted in 1884-85.]

The Determination of the Mechanical Equivalent of Heat by Friction.

Two heavy weights, whose sum is W, in falling through the distance, D, under the force of gravity, g, together heat M grammes of water or their equivalent $T°$ centigrade in a small calorimeter by means of a friction gear applied directly to the axle of the pulley of an Atwood's machine. When the gear is disconnected, the small difference, w, between the two weights is capable of driving the apparatus at the same rate and under the same strain, heating the water only $t°$.

The difference in the amounts of mechanical energy employed is $(W - w)\, g\, D$; dividing this by the corresponding difference in the number of units of heat produced, $M\,(T° - t°)$, corrected if necessary for cooling, we have the number of ergs necessary to heat 1 gramme 1° centigrade, that is, the *mechanical equivalent of heat*,* J, given accordingly by the formula,

$$J = \frac{(W - w)\, g\, D}{M\,(T° - t°)}$$

* The true value probably lies between 41.6 and 41.9 megergs per gramme degree centigrade.

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IMPACT AND CRUSHING.

[Omitted in 1884-85.]

The Determination of the Mechanical Equivalent of Heat by Impact and Crushing.

Two heavy bars are suspended horizontally, end to end, by means of two chains, then drawn apart and held each by a powerful electro-magnet at a measured height above its normal level. The common circuit being broken, a piece of lead is crushed between them by the force of impact, and dropped immediately into water. The rebound, which is slight, is estimated by means of a graduated scale, and the weight and rise of temperature of the lead and water are found by the ordinary methods. It is then easy to calculate the loss of mechanical energy and its equivalent in heat.*

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* See Trowbridge, Experiment 105.

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RESISTANCE TUBE.

[Omitted in 1884-85.]

The Determination of the Mechanical Equivalent of Heat by means of a Resistance Tube.

Water under high pressure is driven through a galvanized iron pipe packed with resisting material; the loss of pressure is measured with a manometer and divided by the corresponding rise in temperature to find the mechanical equivalent in question. The error due to loss of heat, reduced to a mininum by the proper insulation and by the long continuance of the experiment, may be eliminated by repeating with a diminished flow caused by increased resistance. The difference in temperature is measured by two thermo-electric junctions of German silver, one on each side of the resistance, and connected with a delicate galvanometer, whose graduation is the object of a separate experiment.

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CALIBRATION OF THERMOMETER.

The Calibration of a Thermometer and its Use in the Direct Determination of the Dew-point.

I. The fixed points are determined by surrounding the bulb and as much of the stem as possible first with melting snow or ice, the temperature of which is 0°, then with the steam arising from boiling water, the temperature of which is nearly 100°, but is found more exactly for a given pressure of the barometer by consulting the following table.

Cm.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
72.	98.49	98.53	98.57	98.61	98.65	98.69	98.72	98.76	98.80	98.84
73.	98.88	98.92	98.95	98.99	99.03	99.07	99.11	99.14	99.18	99.22
74.	99.26	99.29	99.33	99.37	99.41	99.44	99.48	99.52	99.56	99.59
75.	99.63	99.67	99.70	99.74	99.78	99.82	99.85	99.89	99.93	99.96
76.	100.00	100.04	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
77.	100.36	100.40	100.44	100.47	100.51	100.55	100.58	100.62	100.65	100.69

The corrections at 0° and 100° are then found by subtracting the actual from the theoretical readings. The tenths of degrees are estimated by the eye,* making sure that the thread of mercury

* For greater exactness, the cathetometer is often employed.

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is unbroken. Care must be taken not to let the boiling water touch or spatter the bulb of the thermometer, to shield the latter from radiation on all sides, and to subject it only to gradual changes of temperature.

II. By manipulation* of the microscopic bubble of air which usually remains in a thermometer, a column of mercury nearly 50° in length is now broken off, and placing the lower end at the melting, or first fixed point, the reading of the upper end is taken, also that of the lower end when the upper is at the second fixed point, whose corrected reading is 100°. The mean is the middle point of the thermometer, and should coincide with the 50° mark.

A column 25° in length is now detached, and using the middle point in combination with the two fixed points, the first and third quarter points are determined in precisely the same way as before. It will be found in general more convenient as well as more precise to set one end of the thread of mercury not exactly at the fixed or middle point in question, but at the correspond-

* See Kohlrausch, page 57, § 23. *Separation of a Thread of Mercury of any desired length.*

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ing mark upon the thermometer, in which case, the necessary correction must be applied.*

The determination of the eighth and sixteenth points is not generally warranted by the precision of the instrument.

III. Snow, water and salt are now mixed in a calorimeter until the lightest possible film of moisture appears upon its polished surface.† The temperature is carefully determined, also that at which the film, upon the addition of more water, gradually disappears. By successive repetitions, the dew-point is determined between two limits, which should not differ by more than 1° from the mean, the corrected value of which may be taken as the true dew-point, and is compared with the indication of the hygrodeik hitherto employed.

*A short table of corrections will be found convenient when numerous thermometric determinations are to be made. The interpolated values are best found for every 5 or 10 degrees upon the thermometer.

† Care must be taken not to breathe upon the surface, nor to approach it too closely with the hand.

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EXAMPLE.

I.	Reading of thermometer at 0°	+0°.4
	Correction " "	−0°.4
	Reading " in steam	100°.0
	Temperature of steam at 77.1 cm.	100°.4
	Correction at 100° . . .	+0°.4
II.	Length of long column, from 0° to	50°.6
	" " " " 100° to	49°.8
	Would have reached from the true zero, + 0°.4, to . . .	51°.0
	— from the corrected 100°, 99°.6, to	49°.4
	Middle point, ½ (51°.0 + 49°.4) .	50°.2
	Length of short column from 50° to	25°.6
	" " " " " 0° "	24°.5
	Would have reached from 50°.2 "	25°.8
	" " " " +0°.4 "	24°.9
	First quarter point . .	25°.35
	Third quarter point, same way,	74°.9

III.	Moisture deposited.	Moisture disappears.
	+8°.3	+10°.2
	7.5	9°.9
	7.9	9°.5

Mean	+8°.9
Correction	−0°.4
Dew-point	+8°.5
Indication of hygrodeik . .	+13°.3

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BOILING POINTS.

[Omitted in 1884-85.]

The Determination of Boiling Points under Atmospheric Pressure.

By means of an apparatus similar to that used in testing a thermometer, the boiling points of many liquids may be determined. The vapor is passed into a tube cooled by a current of water, and the liquid formed by the condensation is allowed to flow back into the boiler — a precaution of especial importance in the case of a mixture, the more volatile portion of which would otherwise escape. The pressure in the tube is maintained at 76 centimetres by connecting the free end with a gasometer to which the necessary weight or counterpoise has been attached.

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VAPOR TENSION.

[Omitted in 1884-85.]

The Determination of the Tension of the Vapors of Liquids at different Temperatures by means of Regnault's Apparatus.

Substituting a strong receiver for the gasometer used in the determination of boiling points under atmospheric pressure, it is possible to determine the boiling point of a liquid under a great variety of pressures, or what is the same thing, the tension of the vapor at different temperatures. The air in the receiver is exhausted or compressed by an ordinary air-pump, and the pressures are determined by a mercury gauge or air-manometer.

The tension of vapors at different temperatures may also be determined by Gay-Lussac's method, for which see under Vapor Density.

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METALLIC THERMOMETER.

[Omitted in 1884-85.]

The Determination of Difference of Expansion by means of the Metallic Thermometer.

Two rods are bound together at one end, and at the other carry an index jointed to each and armed with a mirror in connection with a telescope and scale, so that the relative motion of the free end may be thus indefinitely multiplied; upon heating the thermometer, thus constructed, the index and mirror are observed to move through a certain angle, from which the difference of expansion may be easily calculated. Knowing the law of expansion of one of the rods, that of the other is determined.

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LINEAR EXPANSION.

[Omitted in 1884-85.]

The Direct Measurement of the Coefficient of Linear Expansion of a Solid.

A horizontal bar, about one metre long, is surrounded with melting ice, which is prevented from covering two fine scratches, one near each end, by means of short water-tight tubes at right angles to the bar, through which the position of the scratches may be read. The distance between them must be accurately determined by means of a comparator or dividing engine, and the bar being then surrounded by a jacket of steam or other vapor whose temperature is known, new readings are made in the same way, by which the average expansion per unit of length for 1° may be determined through various ranges of temperature. The result is called *the coefficient of linear expansion* of the solid.

The shaft of the reading instrument must be maintained at a constant temperature throughout the experiment.

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VOLUME EXPANSION.

[Omitted in 1884-85.]

The Determination of the Absolute Expansion of Mercury by Gay-Lussac's Method.

The mercury is contained in a U-tube, one branch of which is maintained at 0° by melting ice, the other is heated to various temperatures by means of a jacket of steam or other vapor generated under different pressures as in the determination of vapor tensions. The ratio of the vertical lengths of the two columns in equilibrium, measured with the greatest exactness by the cathetometer, determines the absolute expansion of a given volume of mercury, which is the basis of the most accurate determinations of the expansion of solids.

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HYDROSTATIC METHOD.

[Omitted in 1884-85.]

The Determination of the Expansion of a Solid by Weighing in a Liquid.

The displacement of a solid is equal to the loss of weight in any fluid divided by the density of the latter, which may be determined at a given temperature by the specific gravity bottle, or better by Gay-Lussac's method. In the case of water it may be taken directly from the tables. The loss of weight is determined by the hydrostatic balance, suspending the solid by a fine thread attached to the stirrup beneath one of the pans, and passing through a small hole in the bottom of the balance-case and table so that the vapor may not corrode the instrument. Knowing the displacement at different temperatures, the law of expansion is determined.

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EXPANSION OF GLASS.

[Not required in 1884-85.]

The Determination of the Expansion of Glass by means of the Weight Thermometer.

The bulb of the thermometer whose empty weight* has previously been determined is filled with mercury at 0° † by surrounding with melting snow or ice, and is then placed in boiling water; the mercury expelled is carefully collected and weighed, and after the mercury which remains has cooled to the temperature of the room, the bulb with its contents is re-weighed. The quotient of the weight expelled, w, by that which remains, W, is nearly equal to the difference between the cubical expansions of mercury and glass for the given range of temperature. The latter is determined by the thermometer which has already been calibrated, and knowing the expansion of mercury, 0.018153 for 100°, the cubical expansion of glass is roughly determined, from which e, the average coefficient of expansion for 1°, is approxi-

* It is not necessary to distinguish between true and apparent weights in this experiment.

† Pickering, *Physical Manipulation*, Volume II, page 76.

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mately found by dividing by the number of degrees.*

Strictly, however, the last two results should be multiplied by the ratio of the weight of mercury remaining to the original weight; for if W grammes of mercury at 100° occupy a greater space than $W + w$ grammes at 0° in the ratio of the expansion of the glass vessel, $1 + 100\,e$, the volume of a given weight of mercury must increase in the ratio,

$$\frac{W + w}{W} \times (1 + 100e),$$

which should be equal to 1.018153, whence

$$e = \frac{W}{W + w}\left(0.00018153 - \frac{w}{100\ W}\right),$$

in which we must substitute the actual range of temperature, provided that it be greater or less than 100°, and the corresponding average value of the coefficient of expansion of mercury, not differing perceptibly, in the experiment, from 0.00018153.

Since the mercurial and absolute air thermometers agree at 0° and 100°, the result is nearly independent of the difference between them.

* This coefficient varies, according to the nature of the glass, from .00002 to .00003.

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Example.

Weight of mercury expelled .	5.320 g.
" " bulb with mercury	400.0 g.
" " bulb with air . .	50.0 g.
" " mercury remaining .	350.0 g.
Number of degrees . . .	100.0
Difference of expansion for 100°.0 between mercury and glass, nearly,	0.01520
Expansion of mercury for 100°.0	0.01815
Expansion of glass for 100°.0, nearly .	0.00295
Average coefficient of expansion for 1° between 0° and 100°, nearly	0.0000295
True value (.0000295 × 350. ÷ 355.32),	0.0000291

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EXPANSION OF SOLIDS.

[Omitted in 1884-85.]

The Determination of the Expansion of a Solid by means of the Weight Thermometer.

The solid, whose mass and displacement must be known, is enclosed in the thermometer bulb; the additional weight of mercury necessary to fill the empty space is determined, and its volume calculated by referring to a table of densities. The capacity of the bulb is then equal to the sum of the displacements of the solid and mercury, from which the capacity at any other temperature may be calculated if the expansion of the glass is known, and subtracting the volume of the mercury remaining, calculated in the same way as before, we have the displacement of the solid at a new temperature, from which the expansion is determined. A more accurate result may be obtained by forming a thermometer bulb as in the case of glass out of the substance whose expansion is to be determined. Neither method is applicable to solids soluble in mercury.*

* In these cases, the hydrostatic method is preferable, and in no case should t be attempted to use other liquids in the weight thermometer.

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EXPANSION OF LIQUIDS.

The Determination of the Coefficient of Expansion of a Liquid by means of the Specific Gravity Bottle.

A narrow-mouthed glass-stoppered bottle is filled with alcohol, and surrounded with hot water up to the neck. As soon as the thermometer, with which the alcohol is stirred, ceases to rise, the bottle is corked, removed from the bath, carefully dried and weighed. In this way the density is determined for various temperatures between that of the room and the boiling point, exactly as in a previous experiment, and the density at 0°, D_0, is calculated by inspection of the differences of the series. At any two temperatures, T_1 and T_2, let the densities be D_1 and D_2, then the average coefficient of expansion, e, for 1° between T_1 and T_2 is given by the expression,

$$e = \frac{D_1 - D_2}{T_2 - T_1} \times \frac{D_0}{D_2 D_1},$$

for this may also be written

$$e = \frac{\frac{1}{D_2} - \frac{1}{D_1}}{(T_2 - T_1)\frac{1}{D_0}},$$

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and remembering that for the volume, V, of the unit of mass, we have $V_0 = \frac{1}{D_0}$; $V_1 = \frac{1}{D_1}$; $V_2 = \frac{1}{D_2}$; it follows that

$$e = \frac{V_2 - V_1}{(T_2 - T_1) V_0},$$

which is by definition the average coefficient of expansion from T_1 to T_2.

It will be sufficiently exact to neglect the corrections for buoyancy and for the change of capacity of the bottle, and instead to add the coefficient of expansion of glass, .00003 nearly, to the apparent value of the coefficient of expansion of the liquid. The results are tabulated and compared with the value deduced for alcohol of the given density, from the table already employed.

The principal sources of error in this experiment are due to inequalities of temperature, its change while corking, evaporation of alcohol and absorption of aqueous vapor from the atmosphere. The results are of course affected by the differences between the mercurial and absolute air thermometers; but if the instrument be made of common glass, these differences will be small compared with the errors of observation.

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EXAMPLE.

Temperature.	Density.	Difference.	Difference for 1°.
75°	.7688	.0141	.00094
60°	.7829	.0180	.00090
40°	.8009	.0172	.00086
20°	.8181		

Difference for 1° from 0° to 20°,
estimated (94 — 90 — 86 — 82) . .00082

Density at 0°, estimated
.8181 + 20 × .00082 = ...8345

Per cent. of alcohol, density .8181
at 20°, from table . . . . . 90.00 %

Coefficient of expansion:—

60° to 75°, $\frac{.7829 - .7688}{15} \times \frac{.8345}{.7829 \times .7688} + .00003 = .00133$

40° to 60°, $\frac{.8009 - .7829}{20} \times \frac{.8345}{.8009 \times .7829} + .00003 = .00123$

20° to 40°, $\frac{.8181 - .8009}{20} \times \frac{.8345}{.8181 \times .8009} + .00003 = .00113$

0° to 20°, $\frac{.8345 - .8181}{20} \times \frac{1}{.8181} + .00003 = .00103$

16° to 21°, $\frac{.82163 - .81727}{5} \times \frac{.8345}{.82163 \times .81727} = .00108$*

---

* For 90 per cent. alcohol, from table:—

Temperature.	Density.	Temperature.	Density.
15°	.8225	20°	.8181
16°	.8216	21°	.8173
17°	.8208	22°	.8164
3)48°	3)2.4649	3)63°	3)2.4518
16°	.82163	21°	.81727

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EXPANSION OF GASES.

[Omitted in 1884–85.]

The Determination of the Coefficient of Expansion of a Gas.

The density of a gas is determined at two temperatures, 0° and 100° for instance, by means of a specific gravity flask, exactly as in a previous experiment, and the coefficient of expansion is calculated by the same formula as in the case of a liquid, remembering to add the coefficient of expansion of the glass of which the flask is composed. The weighings are made after the flask has had time to reach the temperature of the room, and the temperature is calculated at which the volume of the gas would be reduced to nothing, that is, the *absolute zero*, which has been estimated by different authorities from 272 to 273 degrees below the freezing-point on the centigrade scale.

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TEMPERATURE AND PRESSURE.

[Omitted in 1884–85.]

The Determination of the Coefficient of the Increase of Pressure with the Temperature by means of the Absolute Air Thermometer.

The pressure of air in a receiver is determined at 0° and at 100° by means of a mercury manometer, whose capacity should be small in comparison with that of the receiver. The increase of pressure under constant volume is then found, allowing for the expansion of the air due to the change in the level of the mercury and to the increased capacity of the receiver, the latter being calculated from the coefficients of expansion and of elasticity of the material of which the receiver is made. The increase of pressure is divided by the corresponding rise in temperature (100°) and by the pressure at 0° to find the coefficient in question, which is almost invariably greater than the coefficient of expansion under constant pressure, owing to the departure of gases from the law of Boyle and Mariotte. Finally the temperature is calculated at which air would exert no pressure, and compared with the absolute zero found from the coefficient of expansion.

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METHOD OF MIXTURE.

The Determination of the Specific Heat of a Liquid by the Method of Mixture.

By mixing known weights of two liquids at given temperatures and observing the temperature of the mixture, the specific heat is determined.

A kilogramme of mercury is heated in an iron cup, surrounded by steam, to 100°, and mixed in an insulated vessel* with water from melting ice, previously weighed in an eight-ounce bottle, until a delicate thermometer after brisk agitation shows that the temperature of the mixture is the same as that within the vessel before the experiment which should agree with that of the room. The bulk of the two liquids should be poured into the vessel as nearly as possible at the same time, the water preferably first, then a little less mercury by volume, reserving a portion of each to bring about the exact adjustment of temperature.

Since the temperature of the vessel is unchanged, we assume that no heat has been received or given up by the mixture as a whole, whose rate of cooling is, therefore, nil.

* The vessel may be of iron, glass, or any material not attacked by mercury or water. The sides should be as thin as is consistent with the required strength.

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Knowing the weights of the bottle and vessel before and after the mixing, we find that of the water and that of the mercury used in the experiment. Multiplying the latter by the number of degrees which the mercury has fallen, we have the number of gramme-degrees of heat lost by the mercury; multiplying the number of grammes of water, originally at o°, by the temperature of the mixture, we have the equivalent in gramme-degrees of water, that is, the number of units of heat gained by the water. The quotient of the latter by the former is the value of one gramme-degree of mercury, that is, its mean *specific heat* referred to water for the given ranges of temperature.

The chief errors in this experiment are from cooling in pouring the mercury, and warming in the case of water; also from unequal distribution of temperature in the mixture of water both with ice and mercury. A cork is provided with a small hole so that the mercury may be poured directly from the heater; the ice and water must be surrounded with non-conducting material, and briskly agitated before pouring. Care must, of course, be taken to prevent small fragments of ice from following the stream.

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Example.

Temperature of room . . .	22°.72
Temperature of mixture . .	22°.70
Weight of bottle with ice and water .	504.75 g.
" " " " remainder .	427.75 g.
" " water at 0° . . .	77.00 g.
" " vessel with mixture .	1120.0 g.
" " empty vessel . . .	364.0 g.
" " mercury and water .	756.0 g.
" " water, brought down . .	77.0 g.
" " mercury at 100° . .	679.0 g.
Fall of temperature of mercury . .	77°.30
Number of gramme-degrees of mercury (77.3 × 679.) .	5249 × 10
Number of gramme-degrees of water (22.7 × 77) . .	1748
Mean specific heat of mercury between 22°.7 and 100° referred to water between 0° and 22°.7,	
1748 ÷ 52490 = . . .	.0333

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THERMAL CAPACITY OF THERMOMETER.

The Determination of the Specific Capacity of a Thermometer for Heat.

The bulb of a thermometer is just covered with mercury in a miniature calorimeter, and the mercury is weighed and allowed plenty of time to cool, in a separate vessel, to the temperature of melting ice, while the thermometer is heated, after several trials, enough to bring the cold mercury to the normal temperature of the calorimeter. When the thermometer in falling has reached the desired mark, it is plunged into this mercury which, just before, has been transferred to the calorimeter with the greatest possible speed. Stirring by twisting the stem of the thermometer, the resulting temperature is observed, and should not differ perceptibly from that of the calorimeter previously determined.

Multiplying the number of grammes of mercury by the number of degrees which they have risen, we divide the product by 30 to find the equivalent in water units; then dividing by the number of degrees which the thermometer has fallen, we find how many units of heat are given out in falling one degree, that is, the *specific capacity* of the thermometer for heat.

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This specific capacity is definite only when a very small portion of the stem adjoining the bulb is heated or cooled in each case; but from the mean of several results obtained in this way, an average value may be determined, accurate to at least two places of decimals.

EXAMPLE.

I.	Temperature of mercury .	+2°.0
	" " thermometer*	40°.0
	" resulting . .	25°.0
	Weight of mercury . .	5.60 g.
	Rise of temperature . .	23°.0
	Product	128.8
	Reduced to water units . .	4.29
	Fall of thermometer . .	15°.0
	Specific capacity (4.29 ÷ 15)	.286
II.	Result, same way, . .	.299
III.	" " " . .	.295
IV.	" " " . .	.280
V.	" " " . .	.265
VI.	" " " . .	.248
VII.	" " " . .	.258
VIII.	" " " . .	.278
IX.	" " " . .	.278
X.	" " " . .	.306
Mean of 10 observations	. .	.279

* Number 39,738.

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THERMAL CAPACITY OF VESSEL.

[Not required in 1884-85.]

The Determination of the Specific Capacity of a Calorimeter for Heat.

A large calorimeter, whose empty weight is known, is filled with hot water at an observed temperature of 50° or more, then suddenly emptied and refilled with cold water from a weighed vessel. The temperature of this water is found, before and after pouring, by means of a delicate thermometer. The initial temperature should be so chosen that the final may not differ perceptibly from that of the room. A second thermometer is placed in the hot water, the two instruments being alternately observed at intervals of one minute for some time previous to pouring, stirring the water with the stem of the thermometer in each case. It is impossible to make exact corrections for the gain or loss of heat under the varying conditions of the experiment; but it is best to assume that the rates of

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change are constant, taking care to replace the hot water by cold as quickly as possible, so as to reduce this error to a minimum.

The cold water vessel and calorimeter are finally reweighed, so that the amount of cold water used is evidently determined, together with that of the hot water film clinging to the sides of the calorimeter.

The total thermal capacity of the hot bodies is to that of the cold ones inversely as their respective changes of temperature. The former is easily calculated, neglecting the heat absorbed by the thermometer; and subtracting the specific capacity of the hot water film, we have that of the calorimeter alone.

The thermal capacity of a calorimeter is definite only provided that the whole or a given portion participates in every case in the changes of temperature in question. The method is especially applicable to calorimeters of such construction that the thermal capacity can not directly be calculated from the specific heat and quantity of the materials of which they are composed.

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EXAMPLE.

Weight of calorimeter, empty .	3327 g.
" " cold water used . .	3695 g.
Total	7022 g.
Weight after experiment . .	7032 g.
Weight of hot water film . .	10 g.

No. of Minutes.	Temperature of Calorimeter.	Difference.	Temperature of Cold Water.	Difference.
1	89°.4			
2	——	1°.7	16°.25	
3	87°.7		——	.03
4	——	1°.6	16°.28	
5	86°.1		——	.02
6	——	1°.6	16°.30	
7	84°.5			

Temperature after experiment . . 18°.40

Calculated temperature of calorimeter at time of pouring (8th minute),

$84°.5 - \frac{1}{2} \times 1°.6 =$ . . . 83°.7

Calculated temperature of cold water at 8th minute, $16°.30 + .02 =$. 16°.32

Cooling of calorimeter, $83°.7 - 18°.4 =$ 65°.3

Rise of temp. of water, $18°.40 - 16°.32 =$ 2°.08

Thermal capacity of calorimeter and film,

$$\frac{2.08}{65.3} \times 3695 = \text{. . . . .} \quad 118.$$

Weight of hot water film . . . 10.

Specific capacity of calorimeter for heat, 108.

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SPECIFIC HEAT OF SOLIDS.

The Determination of the Specific Heat of a Solid by the Method of Mixture.

The same method may be employed as in the case of liquids, but to avoid the increased error due to cooling in transferring the solid from the heater to the calorimeter, the following is most frequently adopted.

A brass calorimeter, highly polished, of one or two deciliters capacity and known weight, is insulated from an outer vessel by pointed cork supports, and filled with 550 or 1100 grammes of hot lead shot, whose temperature is observed every half minute by means of a thermometer which penetrates the cork tightly closing the mouth of the calorimeter. The shot should be briskly shaken between the observations, which are discontinued at a given signal from the clock as soon as the rate of cooling has been sufficiently determined, and at the next stroke of the clock, water from melting ice is poured with the

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greatest care and rapidity down the cold stem of a second thermometer used to stir the ice and water in the bottle, which must, of course, be weighed before and after the experiment. The calorimeter and its contents are exposed as little as possible to the air, and should be carefully corked and stirred or shaken if possible before the next signal is heard, after which the observations of temperature are resumed, and carried on until the temperature, which should not differ greatly from that of the room, changes by an amount which may be attributed to that difference.*

The temperature of the shot and of the mixture must be reduced to the time of mixing, determined by the signal which occurs in the middle of the process; that of the water is of course o°; hence the number of units of heat transferred from the hot bodies to the water is found by multiplying the weight of the latter by the temperature of the mixture. Dividing by the number of degrees which the thermometer has fallen, we find the total thermal capacity of the hot bodies, from which deducting that of the

* The rate of cooling is approximately proportional to the difference of temperature between the inner and outer vessel, and inversely as the total thermal capacity of the calorimeter with its contents.

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calorimeter* and thermometer, we have that of the shot, which finally divided by their weight gives the specific heat in question. This is then compared with the value tabulated for pure lead.†

EXAMPLE.

I. Weight of water-bottle . . 518.7 g.
Ditto after experiment . . 477.1 g.

Weight of water at 0° . . . 41.6 g.
" " lead shot . . . 550 g.

Temperatures every half minute:

No.	Before mixing.	Dif.	No.	After mixing.	Dif.	No.	After mixing.	Dif.
1	71.0		15	——		29	20.6	—
2	70.0	1.0	16	——	—	30	.6	—
3	69.0	1.0	17	15.3	—	31	.6	—
4	68.0	1.0	18	18.3	—	32	.7	—
5	67.1	.9	19	19.3	—	33	.7	—
6	66.3	.8	20	19.8	—	34	.7	—
7	65.5	.8	21	19.9	.1	35	.7	—
8	64.7	.8	22	20.0	.2	36	.7	—
9	64.0	.7	23	20.2	.1	37	.7	—
10	63.2	.8	24	20.3	.1	38	.8	—
11	62.4	.8	25	20.4	.0	39	.8	—
12	61.7	.7	26	20.4	.1	40	.8	—
13	61.0	.7	27	20.5	.0	41	.8	—
14	——	—	28	20.5	.1	42	.8	—
15	——	—	29	20.6		43	.9	—

*For a brass calorimeter, the thermal capacity is very nearly equal to ninety-four thousandths of its weight.

†In repeating the experiment, pure copper shot may be used instead of the lead.

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Temperature at 30th signal	20°.6
" " 40th "	20°.8
" " time of mixing, (15th signal) calculated, $(20°.6 - \frac{15}{10} \times (20°.8 - 20°.6))$	20°.3
Temperature of shot $(61° - 2 \times 0°.7)$	59°.6
Units of heat transferred (41.6×20.3)	844.5
Thermal capacity of hot bodies, $844.5 \div (59.6 - 20.3)$	21.49
Weight of calorimeter	46.0 g.
Thermal capacity $(46.0 \times .094)$	4.32
" " of thermometer	.28
" " of shot, $(21.49 - 4.32 - .28)$	16.89
Specific heat of shot $(16.89 \div 550)$	.0307
Value tabulated for pure lead	.0314

II. Weight of copper shot	400 g.
Temperature of copper shot	74°.2
Weight of water at 0°	44.5 g.
Temperature of mixture	35°.9
Units of heat transferred	1598.
Thermal capacity of hot bodies	41.72
Calorimeter and thermometer	4.60
Thermal capacity of shot	37.12
Specific heat of shot	.0928
Value tabulated	.0951

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REGNAULT'S APPARATUS.

[Reserved in 1884-85.]

The Determination of the Specific Heat of a Solid by Regnault's Apparatus.

A wire gauze basket containing the solid is heated within a steam drum to a temperature in the neighborhood of 100°, measured by a thermometer especially adapted to this purpose, then suddenly lowered through two trap doors into a quantity of cold water contained in a calorimeter of the ordinary sort, separated, however, from the steam drum by a jacket of cold water. The contents of the calorimeter are agitated by raising and lowering the basket by means of the pulley from which it is suspended, and the temperature is accurately determined. The thermal capacity is calculated in the usual way, that of the basket being, of course, deducted, and the remainder is divided by the weight of the solid in question in order to find its specific heat.

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HEAT OF FUSION.

The Determination of the Latent Heat of Water.

The calorimeter used to determine the specific heat of a solid is half filled with hot water, the temperature of which is observed every half minute, then cooled to the temperature of the room by a weighed quantity of melting ice, whose bulk, with that of the water, should be nearly sufficient to fill the calorimeter. The ice must be carefully dried upon the loose cotton separating it from the balance pan, protected, in transferring, from the heat of the hand, and rapidly stirred until the last particle has disappeared. The thermometer is read every half minute until a constant temperature is attained.

The excess of temperature over that of the room is nearly proportional to the rate of cooling; bearing this in mind, the temperature of the hot water and, if necessary, that of the mixture, may be reduced to the time of disappearance of the ice. Calculating as usual the number of units of

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heat given out by the water, we divide by the weight of ice to find the heat absorbed by one gramme of that substance; then subtracting the number of units required to raise a gramme after melting to the temperature of the mixture, numerically equal to that temperature, we have the number of units spent in the fusion of one gramme of ice, that is, the *latent heat of water.*

EXAMPLE.

Temperatures at intervals of half a minute:

No.	Before mixing.	Dif.	No.	After mixing.	Dif.
1	87°.8		6	27°.8	
		1.2			2.4
2	86°.6		7	25°.4	
		1.1			1.3
3	85°.5		8	24°.1	
		1.1			0.1
4	84°.4		9	24°.0	
					0.0
5	——		10	24°.0	

Temperature of room . . . . . 23°.4

Number of interval.	Average excess of temperature over that of room.	Correction for rate of cooling.
4–5	60°	—1.0
5–6	30?	—0.5
6–7	3.2	—0.1
7–8	1.3	—0.0

Total correction . . . . . —1.6

Temperature at 4th signal . . . . . 84°.4

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Temperature reduced to 8th signal . .	82°.8
Temperature of mixture at 8th signal .	24°.1
Fall of temperature	58°.7
Weight of calorimeter with mixture .	124.40
" " calorimeter	46.00
" " ice	30.00
" " water (124.4 — 46 — 30) .	48.40
Thermal capacity of calorimeter and thermometer	4.60
Total thermal capacity with water . .	53.00
Units of heat lost (53 × 58.7)	3111
Heat spent on one gramme of ice (3111 ÷ 30)	103.7
Heat spent between 0° and 24°.1 . . .	24.1
Latent heat of water	79.6
Value according to Regnault	79.0
" " " Bunsen	80.0

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HEAT OF SOLUTION.

[Not required in 1884-85.]

The Determination of the Latent Heat of Solution in Water.

The method is the same as that adopted for the latent heat of fusion, only substituting for the ice, at 0°, a comparatively small weighed quantity of a salt at the temperature of the room and of the mixture, so that the usual corrections may be omitted.

The heat of solution, determined in this way, strictly includes the heat of dilution, so that the result is definite, only, provided that the quantity of water used is indefinitely great.

EXAMPLE.

Weight of nitrate of ammonia . . .	3.00 g.
" " water	70.20 g.
Temperature of water, reduced to time of mixing	23°.00
Temperature of mixture	20°.00
" " room	20°.00
Capacity of water, calorimeter, etc. (70.2 + 4.8)	75.00
Fall in temperature	3°.00
Units of heat given out by water . .	225.
Latent heat of solution (225 ÷ 3) . .	75.0

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HEAT OF COMBINATION.

[Omitted in 1884-85.]

The Determination of the Heat of Chemical Combination of Solids or Liquids by means of a Calorimeter.

A small quantity of zinc is dissolved in a known amount of dilute sulphuric or muriatic acid in a glass calorimeter, and the rise of temperature accurately determined. An equivalent quantity of oxide of zinc* is dissolved in the same quantity of acid, and the change of temperature measured as before.

Calling the specific heat of the dilute acid or solution the same as that of water, the energy of combination of zinc with oxygen is approximately calculated from the difference in the heat generated in the two cases, remembering to add the heat of combustion of the equivalent of hydrogen set free.

*0.652 g. of zinc is here equivalent to 0.812 g. of oxide of zinc, and sets free 0.020 g. of hydrogen. The heat of combustion of hydrogen may be taken as 34,000 gramme-degrees. 50 g. of dilute acid should be raised about 3°.4 more in one case than in the other, since the heat of combustion of zinc is about 1300.

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HEAT OF VAPORIZATION.

The Determination of the Latent Heat of Steam.

A brass calorimeter is nearly filled with a weighed quantity of water, and several observations of the temperature, which should be as low as possible, are made at intervals of half a minute each; then steam from a copper generator is passed rapidly but steadily into the water until the temperature is as much above that of the room as it was, before, below it, so that there may be no correction for cooling during the interval. The steam being now suddenly cut off, the temperature at the instant is calculated from several more readings of the thermometer, and the rise of temperature from the average difference of three or more pairs of observations at corresponding intervals just before and just after the process.

The experiment is rendered more accurate by passing the steam through a trap just above the cork which closes the mouth of the calorimeter, and by attaching a small brass stirrer to the stem of the thermometer, to maintain uniformity of temperature.

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To the weight of water must of course be added the thermal capacity of the calorimeter, thermometer and stirrer; multiplying by the rise in temperature, we have as usual the number of units of heat exchanged during the process, and dividing by the weight of steam, determined by the increase in weight of the calorimeter, we have the heat evolved from each gramme, from which, finally, we subtract the amount given up in cooling from the boiling point* to the temperature of the mixture, numerically equal to the difference in degrees, to find the number of units set free by one gramme of steam in the act of condensation, that is, the *latent heat of steam.*

EXAMPLE.

Weight of water and calorimeter including cork, thermometer and stirrer	130.20 g.
Calorimeter, etc.	63.00 g.
Thermal capacity of water = weight,	67.20
" " " calorimeter . .	4.32
" " " thermometer .	.28
" " " stirrer	.20
Total	72.00

* See Table under Calibration of Thermometer.

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Temperature at intervals of half a minute:

No.	Before admitting steam.	Dif.	No.	While admitting steam.	No.	After admitting steam.	Dif.
1	6.8		7	12.	13	——	—
2	7.0	.2	8	16.	14	37.5	.1
3	7.2	.2	9	21.	15	.4	.2
4	7.3	.1	10	28.	16	.2	.2
5	76.	.3	11	31.	17	.0	.2
6	—	—	12	35.	18	36.8	

Temperature of room . . . . . 23°.0

Calculated temperature at instant when steam was cut off, $37°.5 + .1 =$. 37°.6

Rise of temperature, (1) $37°.5 - 7°.6 =$ 29°9

" " " (2) $37°.4 - 7°.3 =$ 30°.1

" " " (3) $37°.2 - 7°.2 =$ 30°.0

Mean . . . . . 30°.0

Units of heat (72×30) . . . . . 2160.

Weight of calorimeter with mixture . 133.80 g.

" " " " water . . 130.20 g.

" " steam condensed . . . 3.60 g.

Heat evolved by 1 g. of steam,

$2160 \div 3.60 =$ 600.0

Heat given out from 100° to 37°.6 . . 62.4

Latent heat of steam 537.6

Value tabulated, mean 536.3

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SPECIFIC HEAT OF GASES.

[Not required in 1884-85.]

The Determination of the Specific Heat of Air under Constant Pressure by means of a Metallic Worm.

A current of air is passed from a gasometer under a known pressure through a steam jacket, then directly into a coil of lead tube within a calorimeter whose thermal capacity has been calculated or determined.* The coil is covered with a known weight of water, which must be stirred sufficiently during the experiment, and the times are taken when a delicate thermometer passes each division marking a tenth of a degree, beginning with a temperature several degrees below and ending with one as many degrees above the normal temperature of the calorimeter, which, owing to conduction from the steam jacket, will be considerably higher than that of the room. The rate of flow of the air is then found by cut-

* See under Thermal Capacity of Vessel.

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ting off the air supply from the gasometer, and timing its fall though successive litre-divisions.*

In each case the times are combined by the method peculiar to consecutive observations,† so as to find the average rise in temperature and flow in cubic centimetres for one second. The former, multiplied by the thermal capacity of the calorimeter and its contents, gives the heat set free in one second; the latter, multiplied by the density of the air for the given conditions of temperature, pressure and humidity,‡ gives the number of grammes of the air which gives out this heat in falling from 100° § to the average temperature of the mixture.‖ Dividing the amount

* The accuracy of the divisions may be tested by passing air from the gasometer into a graduated litre jar or flask, inverted in a pneumatic trough, so that the water may be displaced by air at the same pressure as that within the gasometer.

† See Example under Simple Pendulum.

‡ The temperature and pressure within the gasometer are found by a thermometer and mercury-gauge which penetrate into the interior, adding of course to the reading of the latter that of the barometer outside. The dew point is found by the condensation of moisture from the air escaping from the worm.

§ In order that the air may be heated exactly to the boiling point of water, which should strictly be found by consulting the barometer, the steam, which is often superheated, must be passed very slowly through the jacket, and the escape valve opened wide to the outer air. The connecting pipes should be carefully covered with insulating material, since otherwise the current of air will cool with great rapidity.

‖ More exactly, the average temperature of the air issuing from the worm, which should be insensibly higher.

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of heat set free by this weight of air, and by the number of degrees through which it fell, we have the amount of heat given out by one gramme of air in falling through one degree, that is, the *specific heat of air under constant pressure.**

SPECIFIC HEAT OF MOIST AIR UNDER CONSTANT PRESSURE.

AFTER REGNAULT AND E. WIEDEMANN.

Dew-point at 76 cm.	Specific Heat.	Dew-point at 76 cm.	Specific Heat.	Dew-point at 76 cm.	Specific Heat.
$-\infty^{\circ}$	.2383	-11°	.2387	$+12^{\circ}$	.2404
-33°	.2383	-10°	.2387	13°	.2405
-32°	.2384	-9°	.2388	14°	.2407
-31°	.2384	-8°	.2388	15°	.2408
-30°	.2384	-7°	.2388	16°	.2410
-29°	.2384	-6°	.2389	17°	.2412
-28°	.2384	-5°	.2389	18°	.2414
-27°	.2384	-4°	.2390	19°	.2416
-26°	.2384	-3°	.2390	20°	.2418
-25°	.2384	-2°	.2391	21°	.2420
-24°	.2384	-1°	.2392	22°	.2423
-23°	.2384	0°	.2392	23°	.2425
-22°	.2385	$+1^{\circ}$	.2393	24°	.2428
-21°	.2385	2°	.2394	25°	.2430
-20°	.2385	3°	.2394	26°	.2433
-19°	.2385	4°	.2395	27°	.2436
-18°	.2385	5°	.2396	28°	.2440
-17°	.2385	6°	.2397	29°	.2443
-16°	.2386	7°	.2398	30°	.2447
-15°	.2386	8°	.2399	31°	.2451
-14°	.2386	9°	.2400	32°	.2455
-13°	.2386	10°	.2401	33°	.2459
-12°	.2387	11°	.2403	100°	.4805

* The pressure is essentially constant in this case, as in Joule's experiment, since the slight decrease in passing through the worm is not spent in external work, but in friction.

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Example.

Thermal capacity of calorimeter, etc. . .	108
Weight of water in grammes	1772
Total thermal capacity	1880
Normal temperature of calorimeter . .	27°.5

Temperatures.	Time.	Temperatures.	Time.	Temperatures.	Time.
26°.00	$17_m\ 47_s$	27°.00	$23_m\ 46_s$	28°.00	$29_m\ 35_s$
.10	18 20	.10	24 14	.10	30 18
.20	18 54	.20	24 59	.20	30 54
.30	19 25	.30	25 35	.30	31 34
.40	20 09	.40	26 09	.40	32 12
.50	20 46	.50	26 35	.50	32 '44
.60	21 24	.60	27 21	.60	33 26
.70	21 57	.70	27 52	.70	33 57
.80	22 33	.80	28 25	.80	34 35
.90	23 12	.90	28 58	.90	35 11
27°.00	23 46	28°.00	29 35	29°.00	35 50
AVERAGE:					
26°.500	$20_m\ 45_s$	27°.500	$26_m\ 41_s$	28°.500	$32_m\ 45_s$

Rise of temperature considered
(28°.5 — 26°.5) . . . . . 2°.000
Time corresponding to ditto
(32 m. 45 s. — 20 m. 45 s.) . . 720 sec.
Heat generated in one second,
1880 × 2 ÷ 720 = ... 5.222

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Litres.	Time.	Litres.	Time.	Litres.	Time.
0	7_m 13_s	10	7_m 53_s	20	8_m 33_s
1	17	11	57	21	36
2	21	12	8_m 1_s	22	41
3	25	13	5	23	45
4	29	14	9	24	49
5	32	15	12	25	53
6	37	16	17	26	57
7	41	17	21	27	9_m 1_s
8	45	18	25	28	5
9	49	19	29	29	9
10	53	20	33	30	13
Av'age 5	7_m 32.9$_s$	15	8_m 12.9$_s$	25	8_m 52.9$_s$

Flow considered, 25—5 litres = 20,000 cu.cm.
Time corresponding,
8m. 52.9 s. — 7m.32.9 s = . . 80.0 sec.
Reading of barometer . . . . 74.53 cm.
Reading of mercury gauge . . . 1.47 cm.
Pressure within gasometer . . . 76.00 cm.
Density of air at 76 cm. 19° and dew-point + 18° 001200
No. of grammes of air delivered in one second, .0012 × 20,000 ÷ 80 = 0.300
Mean of 10 observations in the same way 0.3000
Average fall of temperature, (100°—27.°5) 72°.50
Specific heat of air under constant pressure, 5.222 ÷ .3 ÷ 72.5 = . . .2401
Value tabulated for dew-point + 18° . . .2414

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COMBUSTION OF GASES.

[Not required in 1884-85.]

The Determination of the Heat of Combustion of Coal-Gas in Air.

The experiment is conducted exactly as if determining the specific heat of air, except that the current of air, drawn through the worm by an aspirator, is heated not by steam but by a fine jet of coal-gas, burning within the calorimeter and derived from the gasometer, which is used in the same way as before to determine the rate of flow. Dividing the heat evolved in one second by the corresponding flow in grammes, we have the *Heat of Combustion* of the coal-gas.

EXAMPLE.

	Litres.	Time.	Litres.	Time.	Litres.	Time.
	0	$7_m\ 22_s$	5	$20_m\ 42_s$	10	$34_m\ 3$
	1	10 2	6	23 22	11	36 37
	2	12 36	7	26 0	12	39 24^s
	3	15 22	8	28 40	13	42 2
	4	17 58	9	31 23	14	44 34
Av'age	2	12 40	7	26 1	12	39 20

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Flow considered, 12—2 litres = 10,000 cu.cm.
Time corresponding,
39 m. 20 s. —12 m. 40 s. = . . 1600 sec.
Rate of flow (10,000 ÷ 1600) . . . 6.25 $\frac{\text{cu. cm.}}{\text{sec.}}$
Density of coal-gas observed . . .000500
Rate of flow in grammes per
second, 6.25 × .0005 = . . .003125

Temperatures.	Time.	Temperatures.	Time.	Temperatures.	Time.
22°.0	$54_m\ 30_s$	25°.0	$58_m\ 10_s$	28°.0	$61_m\ 58_s$
23°.0	55 45	26°.0	59 22	29°.0	63 10
24°.0	56 50	27°.0	60 38	30°.0	64 35
AVERAGE.					
23°.00	$55_m\ 42_s$	26°.00	$59_m\ 23_s$	29°.00	$63_m\ 14_s$

Rise of temperature considered, (29—23) 6°.00
Corresponding time,
(63 m. 14 s. —55 m. 42 s.) . . 452 sec.
Thermal capacity of calorimeter and
contents 2100
Heat evolved in one second,
2100 × 6 ÷ 452 = ... 27.9
Heat of Combustion,
27.9 ÷ .003125 . . . . . 8.9×10^3

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COMBUSTION OF SOLIDS.

[Reserved in 1884-85.]

The Determination of the Heat of Combustion of a Solid.

[A weighed quantity of the solid is lighted by electricity and burned in a combustion-chamber within the calorimeter, a constant supply of air or oxygen being maintained, and the products of combustion are drawn through a worm as in the experiment with gases. The quotient of the heat generated by the weight consumed is the heat of combustion of the solid.]

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THERMAL CONDUCTIVITY.

[Not required in 1884-85.]

The Determination of the Absolute Conductivity of a Substance for Heat.

The temperature of a current of water is taken before and after passing through a short steam jacket; from the greater quantity of steam condensed during the flow, the quantity of heat may be found* which passes in one second between the outer and inner surfaces of the water-pipe. The product of this quantity and the distance between the surfaces, divided by their mean area and by their average difference of temperature, gives roughly† the amount of heat which would be conducted between the opposite faces of a unit cube of the given material for a difference in temperature of one degree, that is, the *absolute conductivity for heat.*

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* Since the latent heat of steam is known.

† Since the inner and outer surfaces are not exactly at the temperature of the water or steam.

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RADIATION OF HEAT.

[Reserved in 1884-85.]

The Determination of the Coefficient of Radiation for Blackened Surfaces in Vacuo.

A sphere coated with lamp-black is filled with a weighed quantity of hot water, and suspended in a chamber with blackened walls from which all air should be exhausted. By a delicate thermometer, the rate of cooling is found and reduced to a difference of one degree in temperature between the hot body and the surrounding walls; then multiplying by the thermal capacity of the sphere, and dividing by the area of its surface, we have the amount of heat radiated from one square centimetre of surface for a difference of one degree, that is, the *coefficient of radiation* in question.

The coefficient is said to increase with the temperature, but the law is not sufficiently established.

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SOLAR RADIATION.

[Omitted in 1884-85.]

The Measurement of Solar Radiation by means of the Pyrheliometer.

The plane blackened surface of a vessel filled with mercury is turned alternately toward the sun and toward an unclouded part of the sky, being shaded in the latter case. By means of a delicate thermometer, the algebraic difference is found between the rate of change of the temperature in the two cases, which multiplied by the thermal capacity of the instrument gives the amount of heat received from the sun in one second. This, multiplied by the area of the spherical surface whose radius is the distance of the sun from the earth, and divided by that of the surface of the instrument exposed, gives the total amount of solar radiation.

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THE THERMOPILE.

[Reserved in 1884-85.]

The Determination of the Total Radiation from a Source of Heat by means of the Thermopile.

The sources of heat compared are the sun* and the incandescent lamp. The total radiation of the sun is to that of the incandescent lamp as the square of the respective distances from the nearer face of the thermopile when a delicate galvanometer connected with the latter shows no deflection. Knowing by astronomy the distance of the sun, and by electrical methods the total radiation of the incandescent light, the total radiation of the sun is easily calculated, neglecting the proportion of heat absorbed by the atmosphere.

The result is to be compared with that obtained by the Pyrheliometer, and is of interest as enabling us to form a rough estimate of the temperature of the sun.

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* A candle or gas flame may be compared, instead, with the incandescent lamp. The total radiation, thus determined, will probably be found to be a small portion of the whole quantity of heat generated by the process of combustion.

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THE PHOTOMETER.

The Determination of the Illuminating Power of Different Lamps.

An oil lamp and the gas or candle with which it is to be compared are enclosed each in a blackened box sliding in one of two parallel dark tubes. The weaker light is fixed permanently in the shorter box at a convenient distance, and by means of covers of suitable length, all extraneous light is excluded from the tube. The distance of the stronger light is then adjusted by drawing out the longer box, beneath its cover, until the right and left halves of a small translucent screen, one illuminated by the stronger light only, the other only by the weaker light, appear equally bright to the eye. The head and screen should be covered by a black cloth in this experiment, in order to exclude as much light as possible, and when the adjustment is made, each half of the screen, in turn, will appear the brighter as the eye is moved from side to side.

The position at which this occurs is found from the mean of several determinations, and the in-

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tensities of the two lights are to each other approximately as the squares of their distances from the screen,* to each of which squares should for greater accuracy be added $1\frac{1}{2}$ times the square of the distance from the central partition on account of obliquity and the length of the diagonal.

The weight of the lamp or candle is found before and after the experiment, and the loss is divided by the time to find the amount burned in one second. The weight of gas consumed per second is found by multiplying the flow by the density. From these data we find the ratio of the light given out by equal weights of different substances, and knowing the market price of one gramme of each, one may readily calculate the comparative economy for the purpose of illumination.

* Intensity or quantity of light must be distinguished from brilliancy, which may be defined as the intensity per unit of surface.

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EXAMPLE.

Weight of lamp before experiment .	159.40 g.
" " " after " .	153.90 g.
Oil consumed	5.50 g.
Time corresponding	1000 sec.
Rate of consumption	.0055 $\frac{g.}{sec.}$
Reading of gas-meter at 48 m. 51 s. .	31.0 cu. ft.
" " " " 57 m. 41 s. .	32.0 cu. ft.
Amount consumed, 1 cu. ft. = .	28,316 cu.cm.
Time corresponding	530 sec.
Density of gas, observed . . .	.000500 $\frac{g.}{cu.cm.}$
Rate of consumption,	
$28{,}316 \times .0005 \div 530 =$ . . .	.0267 $\frac{g.}{sec.}$
Ratio of weights consumed	4.85
Distance of oil lamp from screen . . .	60 cm.
" " gas " " " . .	110 cm.
" " either " " partition . .	7 cm.
Ratio of lights, $\frac{110\times110+1\frac{1}{2}\times7\times7}{60\times60+1\frac{1}{2}\times7\times7} =$	3.31
Ditto for equal weights consumed,	
$4.85 \div 3.31 =$	1.47
Cost of 1 kilo. of oil at $0.16 per gallon,	$0.05
" " " gas at	
$2.25 per 1000 cu. ft.	$0.16
Ratio of cost for equal weights . . .	3.2
Relative economy of oil over gas,	
$3.2 \times 1.47 =$	4.7

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INCANDESCENT LAMP.

[Not required in 1884-85.]

The Investigation of the Economy of the Incandescent Lamp.

By means of the photometer, one of the small incandescent lamps may be tested, the electrical resistance of which varies from five to six ohms, in the circuit of a battery of six or eight small Bunsen cells of not over half an ohm resistance each, and with an electromotive force of nearly $1\frac{7}{8}$ volts. The energy spent in the whole electrical circuit in one second is equal to the square of the electromotive force divided by the resistance; and remembering that the ohm is equal to a thousand million, and the volt, one hundred million absolute units, the result is easily expressed in ergs or megergs. This may be compared with the mechanical equivalent of the heat given out by the candle in one second, the heat of combustion being known.*

From the price of candles and from the cost of a horse power† may be found the relative economy of the two lights.

* The energy developed by the combustion of one gramme of paraffine is equivalent to about 380 thousand megergs.

† One horse-power — 7460 megergs per second,—is said to cost, under favorable conditions in the best steam engines, about 1 cent per hour. The proportion of power utilized by a good dynamo-machine may be compared with that utilized in the battery employed, although the cost of voltaic electricity is comparatively great.

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Example.

Ratio of incandescent lamp to light of candle 1.00

Electromotive force, in absolute units,

12 volts = ... 12 hundred million.

Total resistance of circuit, in absolute units,

8 ohms, = ... 8 thousand million.

Energy per second,

$$\frac{1{,}200{,}000{,}000 \times 1{,}200{,}000{,}000}{8{,}000{,}000{,}000, \times 1{,}000{,}000} = 180 \frac{\text{megergs.}}{\text{sec.}}$$

Cost of ditto at $0.01 per horse-power per hour $0.000,000,067

Rate of consumption of candle . . .0018 $\frac{\text{g.}}{\text{sec.}}$

Cost at $0.30 per lb. $0.000,001,2

Ratio of economy of incandescent light . . 18.

Energy from combustion of candle for one second, .0018 × 380,000 = 684 $\frac{\text{megergs.}}{\text{sec.}}$

Ratio of heating for equal lights 3.8

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LOSS OF LIGHT.

[Omitted in 1884–85.]

The Determination of the Loss of Light by Reflection and Absorption.

I. Thin transparent plates of a given material are interposed, in one branch of the photometer, between the screen and one of two equal lights as, for instance, two gas flames fed from a common tube. The distances of the lights are adjusted and the ratio of their intensities calculated in the usual way, and from the number, n, of reflecting surfaces, the proportion of light lost at each is found by subtracting 1 from the n^{th} root of the ratio.

II. Two plates of different thicknesses are now interposed, one in each branch of the photometer, the excess on one side being n centimetres, and the loss in traversing one centimetre is found in the same way as before.

By illuminating the two halves of the screen with colored or with polarized light, the applications of the principle of photometry may be almost indefinitely extended.

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THE SACCHARIMETER.

[Omitted in 1884-85.]

The Determination of the Rotation of the Plane of Polarization by means of a Saccharimeter.

Sodium light, polarized in a given plane by a Nicol prism, is received by an analyzer, which is turned until the field appears perfectly dark, or until two parts of the field, subjected to opposite rotations by interposing pieces of right and left-handed quartz, appear equally bright to the eye. A tube of length, L, containing sugar in solution, is now interposed, and owing to rotation of the plane of polarization, the analyzer must be turned through A degrees to bring about the same adjustment as before.

The measurements should be confirmed, in all cases, by rotating the analyzer through 180°, since the same phenomena should be repeated, and the weight, w, of sugar in a cubic centimetre of the solution is finally calculated by the empirical formula,

$$w = 0.15056 \times \frac{A}{L}.$$

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THE SEXTANT.

[Not required in 1884-85.]

The Determination of Angular Magnitudes by means of the Sextant.

The instrument is adjusted so that only one image is seen in looking at a distant point, and in this position, the zero reading is accurately determined. Subtracting the latter from the reading in any other position, we have directly the angular distance between two objects whose images may be made to coincide. The graduation is tested by repeating measurements with suitable objects completely around the horizon, the sum of the angles being 360°. The angular diameter of the sun is then measured, setting the reflected image first above and then below the direct image; or, instead, the angle subtended by a rod of known length, taking care to have the rod perpendicular to the line of sight. The distance of any point of the rod may then easily be calculated by the principles of trigonometry.

NOTE. — Students desirous of learning the practical use of the sextant in navigation will be assisted in determining latitude, longitude and time.

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REFRACTION OF LENSES.

The Determination of the Principal Focal Length and Index of Refraction of a Lens.

I. A telescope is focussed so that there is no parallax between the cross-hairs and the image of a distant object, and the lens is set at such a distance in front of a screen that a fine line upon the latter can be seen in the same way without changing the focus; since the telescope, thus adjusted, can focus only parallel rays, the distance of the lens from the screen must be that at which rays diverging from a point are rendered parallel, that is, its *principal focal length.*

II. The image of distant objects may, if practicable, be thrown by the lens upon the screen. The distance between the two is that at which parallel rays can be made to meet, which is, again, the *principal focal length.* If the whole lens or the same portion of it be used in both methods, the two results should agree.

III. A lamp is placed at the nearest possible distance from the screen which permits the lens, exactly in the middle, to form a perfect image of the illuminated perforations of an opaque chimney, the border of the image being tinged neither with red nor with blue. This distance divided by 4 gives a third value of the principal focal

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length differing for small lenses only slightly from the former two.

IV. The distance of the lamp is then increased by about 10 centimetres at a time, and in each case are found the two positions in which the lens can form a perfect image of the perforations, together with the corresponding pairs of distances of the lens from the lamp and screen, called *conjugate focal lengths.* The product of any pair of these divided by the sum gives in each case a value of the principal focal length which should agree closely with those previously obtained.*

The index of refraction is, by definition, very nearly equal to unity plus half the mean radius of curvature† of the two sides of the lens divided by the principal focal length found from the average of the results of the four methods above described.

* When a wave-surface passes normally through a bi-convex lens, the central positions are retarded more than the extremes, and hence a new curvature of definite amount is superposed upon the old. The curvature is of course uniform, tends to make the wave concave in front, and has for a result a definite change in the direction of the extreme rays passing through the lens.

The change of direction is evidently the sum of the divergence before and convergence after refraction, and being constant, the sum of the reciprocals of the conjugate focal lengths must also be constant, since these are proportional to the divergence and to the convergence in every case.

The sum of these reciprocals must be equal to twice the value of either, when the two are equal, and must also be equal to the reciprocal of the principal focal length, which is a particular value when one term is equal to zero. From these considerations, the reductions in III. and IV. evidently follow.

† When there is any considerable difference between the two radii of curvature of opposite faces, the reciprocal of the sum of the reciprocals of the diameters of curvature should be used instead. See under The Spherometer.

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Example.

I. Principal focus by telescope, mean of 5 observations, at . . 15.82 cm.

II. Ditto by distant objects, 10 obs. . 15.49 cm.

III. One-fourth of nearest distance of lamp and screen, $\frac{1}{4} \times 62.6$. 15.65 cm.

IV.

	Light to Screen.	Light to Lens.	Lens to Screen.
1.	70 cm.	24.2	45.8
		46.1	23.9
2.	80 cm.	21.7	58.3
		58.2	21.8
3.	90 cm.	20.7	69.3
		69.4	20.6

From 1, $\frac{24.05 \times 45.95}{70} =$ . . 15.79 cm.

From 2, same way . . . . . 15.84 cm.

From 3, " " . . . . . 15.91 cm.

Principal focal length, mean of 6 values . . . . 15.75 cm.

Mean radius of curvature, from spherometer, ... 16.19 cm.

Index of refraction,

$1 + \frac{1}{2}(16.19 \div 15.75) =$ . . . 1.514

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REFRACTION OF PRISM.

[Not required in 1884-85.]

The Determination of the Refractive Index of a Liquid.

A spectrometer is provided with a telescope and collimator, which are first to be set at right-angles. A hollow prism, filled with water, is then mounted upon the revolving plate in the centre of the instrument so that one face reflects the narrow slit of the collimator, illuminated by a sodium flame, into the field of view of the telescope. The position of the prism is determined by the reading of the vernier attached to the revolving plate, after the latter has been adjusted, by means of its tangent screw, so that the vertical cross-hair in the eye-piece of the telescope bisects the image of the slit. By means of the arm which carries the vernier, the prism is now revolved until its second face replaces the first, and a new reading is made in precisely the same manner as before. The difference of the two is the supplement of the angle A, between the two faces in question.

The prism being now removed, the telescope is turned until the slit is seen directly; it is then set by means of its tangent screw and read by the vernier attached. The prism is then replaced and rotated until the collimator, seen through its

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two faces, seems to be displaced as little as possible, and the telescope is pointed at the refracted image of the slit. By means of the revolving plate, the prism is now turned very exactly into the position of minimum deviation, following the apparent motion of the slit by means of the tangent screw of the telescope, which is then set and read as before. The difference of the two readings, which should be confirmed by several alternate settings upon the direct and refracted images, gives the minimum deviation, Δ, of a ray of sodium light in passing through the prism.

The index of refraction of water of the given temperature for light of the given degree of refrangibility in air is then calculated by the formula,*

$$\mu = \frac{\sin \frac{1}{2}(A + \Delta)}{\sin \frac{1}{2} A},$$

* It is easily seen by geometric inspection that, in the position of minimum deviation, $\frac{1}{2}(A + \Delta)$ represents the angle of incidence and $\frac{1}{2}\Delta$ the angle of refraction. This equation, in the case of a prism, defines the index of refraction.

For very small angles it becomes,

$$\mu = 1 + \frac{\Delta}{A},$$

so that in the case of a bi-convex lens of radius of curvature, R, and principal focus, F, at a distance, K, from the centre, where $\Delta = \frac{K}{F}$ and $A = \frac{2K}{R}$, we have

$$\mu = 1 + \frac{1}{2}\frac{R}{F}.$$

The definition of the index of refraction in the case of a prism is therefore consistent with that adopted for lenses.

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and that in vacuo is found by multiplying by 1.00029, the mean index of refraction of air.

The result is in this case to be compared with those tabulated for the line *D* of the solar spectrum.

EXAMPLE.

First position of prism	138° 24′.5
Second " " "	18° 24′.5
Supplementary angle	120° 0′.0
Angle of prism	60° 0′.0
Direct setting upon slit, 3 obs. . .	359° 49′.6
Setting upon refracted image, 2 obs.,	336° 11′.4
Angle of deviation	23° 38′.2

Index of refraction of water at 15° in air,

$$\mu = \frac{\sin \frac{1}{2}(60° + 23° 38'.2)}{\sin \frac{1}{2} 60°}$$

$$= 2 \sin 41° 49'.1 = \quad . \quad . \quad . \quad 1.3335$$

Index in vacuo (1.3335 × 1.00029), .	1.3339
Compare with Fraunhofer's result for 4°,	1.3336
Other observers (see Watt's Dictionary of Chemistry, Vol. III, pages 615, 629-631)	1.3324 to 1.3360

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DISPERSION OF LIGHT.

[Reserved in 1884-85.]

The Determination of the Dispersive Power of a Prism by means of the Spectrometer.

By the use of a glass prism, the refractive indices of two kinds of light are determined, as, for instance, the characteristic extreme red and blue lines of a potassium flame,* or better, the lines *A* and *H* of the solar spectrum. The difference is generally defined as the *dispersion* of the prisms for the two lines in question, and the quotient of the difference by the excess of the mean over unity may be taken as a measure of the *dispersive power* of the prism, which is to be compared with the results tabulated for the same pair of lines with different kinds of glass, and especially those of similar composition and density.

* The blue line may be seen in the flame of the blast lamp, but will probably be invisible with an ordinary Bunsen burner.

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LENGTH OF LIGHT-WAVES.

[Reserved in 1884-85]

The Determination of the Length of a Wave of Light by means of a Diffraction-Grating.

The grating is mounted upon the revolving plate of the spectrometer, with its face perpendicular to the axis of the collimator, and the successive positions are determined at which the several images of the slit, illuminated with monochromatic light, coincide with the vertical cross-hair of the telescope. The average increase in the difference of the distance from the eye of two adjacent lines of the grating in two successive readings is the length of a wave of light of the given degree of refrangibility, and is easily calculated when the number of rulings and the breadth of the plate are known.

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VELOCITY OF LIGHT.

[Reserved in 1884-85.]

The Determination of the Velocity of Light in Air by means of a Revolving Mirror.

A beam of light is received upon a revolving mirror, reflected to a stationary mirror at a great distance, back again to the revolving mirror and thence upon a screen. If the passage of light were instantaneous, it would fall upon a certain spot in a graduated scale, determined by arresting the revolving mirror in any position which makes the spot visible; but when the mirror is made to rotate rapidly, so that the spot of light appears continuous to the eye, it does not occupy the same position, since the mirror turns through a small angle while the light is travelling to the stationary mirror and back. The distance traversed is measured, and the time occupied is to that of a half revolution as the deflection in degrees is to 360, from which we may easily calculate the velocity of light in air.

EXAMPLE.

Distance between mirrors, 50 metres = . . . 5,000 cm.
Time of half revolution at 750 rev. per sec. . .0006666 sec.
Deflection in degrees, 10′ 48″ = ... 0°.1800
Velocity of light, $\frac{2 \times 5{,}000}{.0006666 \times \frac{0.18}{360}} = . . 3.000 \times 10^{10} \frac{\text{cm.}}{\text{sec.}}$

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MAGNETIC PENDULUM.

The Determination of the Directive Force acting upon a Swinging Magnet.

A bar-magnet is suspended horizontally by a long fine thread, which is twisted or untwisted until the oscillations are directed to the magnetic meridian. The time, t, of oscillation is found by observing as many successive turning-points as possible, and since the moment of inertia,* K,

* The moment of inertia which is marked upon each of the compound magnets, has been calculated in the following way: The mass, M, length, L, and breadth, B, being carefully measured in the usual manner, the mass was first corrected by adding the weight of steel necessary to fill the central hole, and subtracting the weight of the brass bolt-heads projecting beyond the surface; both weights being calculated by multiplying the volume by the tabulated density. The remainder is the weight of a bar of rectangular section which has a moment of inertia equal to one-twelfth the product of the mass into the square of the diagonal of the upper or lower surface. To this was added the product of the combined mass of the bolt-heads into the square of their distance from the centre.

The truth of the formula for the moment of inertia of a bar is easily seen by remembering that the moment of inertia of different cross-sections, neglecting their magnitude, increases as the square of their distance from the centre of the bar, so that the average, like that of the cross-section of a pyramid which increases by the same law, will be one-third of the maximum value. Hence the moment of inertia of a bar will be approximately one-third as great as if the whole mass were situated at the extremities. In point of fact, the breadth of the cross-section cannot be disregarded, for the square of the distance of any particle from the axis of rotation is increased by the square of its side-displacement; hence by the same reasoning the correction will be one-third as great as if the whole mass were confined to the two sides. The total is therefore one-third as great as if the whole mass were concentrated at the corners, that is,

$$K = \tfrac{1}{3} M\left(\left(\tfrac{L}{2}\right)^2 + \left(\tfrac{B}{2}\right)^2\right) = \tfrac{1}{12} M(L^2 + B^2), \text{ Q.E.D.}$$

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is known, the directive force,* D ,may be found from a table† especially prepared for the magnet in question, and sufficiently extended to cover all variations in its strength which are likely to occur.

The directive force is the product of the total quantity of magnetism, positive and negative, into the average distance from the centre and into the horizontal component of the earth's magnetism, H, just as in the ordinary pendulum, whose vibration is in a vertical plane, the directive force is the product of the mass, the distance between the centres of gravity and of suspension, and the (vertical) acceleration of gravity. Denoting by M the moment of the magnet, that is, the product of the first two quantities which compose the directive force, we have

$$D = MH.$$

The moment of a magnet cannot be calculated, like that of a pendulum, from a knowledge of its weight and dimensions; for there is no necessary connection between the distributions

* This term is not in general use, and is employed by certain writers in a different signification. See, however, Kohlrausch, Appendix A, page 193.

† These tables should accompany each magnet, being originally calculated by a formula which applies also to the ordinary pendulum, viz.

$$D = \frac{\pi^2 K}{t^2}$$

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of weight and of magnetism; hence the horizontal intensity, H, cannot be directly determined.

The directive force is, however, of importance in connection with the deflecting power measured by the magnetometer, since together these enable us to calculate both the magnetic moment and the horizontal intensity.

Example.

Time of oscillation of magnet, X, . 7.125 sec.
Directive force, from table . . . 49.63 $\frac{\text{g. cm. cm.}}{\text{sec. sec.}}$

TABLE FOR DIRECTIVE FORCE ACTING ON MAGNET X.

Moment of inertia = 255.3

Time of Oscillation.	0	1	2	3	4	5	6	7	8	9
7.0	51.42	51.28	51.13	50.99	50.84	50.70	50.55	50.41	50.27	50.13
1	49.99	49.84	49.70	49.56	49.43	49.29	49.15	49.01	48.88	48.74
2	48.61	48.47	48.34	48.20	48.07	47.94	47.81	47.67	47.54	47.41
3	47.28	47.15	47.03	46.90	46.77	46.64	46.52	46.39	46.26	46.14
4	46.01	45.89	45.77	45.64	45.52	45.40	45.28	45.16	45.04	44.92
5	44.80	44.68	44.56	44.44	44.32	44.20	44.09	43.97	43.85	43.74
6	43.62	43.51	43.40	43.28	43.17	43.06	42.94	42.83	42.72	42.61
7	42.50	42.39	42.28	42.17	42.06	41.95	41.84	41.74	41.63	41.52
8	41.42	41.31	41.20	41.10	40.99	40.89	40.79	40.68	40.58	40.48
9	40.37	40.27	40.17	40.07	39.97	39.87	39.77	39.67	39.57	39.47
8.0	39.37	39.27	39.17	39.08	38.98	38.88	38.79	38.69	38.60	38.50
1	38.40	38.31	38.22	38.12	38.03	37.93	37.84	37.75	37.66	37.57
2	37.47	37.38	37.29	37.20	37.11	37.02	36.93	36.84	36.75	36.66
3	36.58	36.49	36.40	36.31	36.23	36.14	36.05	35.97	35.88	35.80
4	35.71	35.63	35.54	35.46	35.37	35.29	35.21	35.12	35.04	34.96

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THE MAGNETOMETER.

The Measurement of the Deflection of a Compass-Needle due to a Magnet in a given position.

The magnetometer is levelled and turned so that the needle points as nearly as possible, one end to 0° and the other to 180°. The magnet, whose time of oscillation should have been determined in the immediate neighborhood, is then mounted upon a pivot to the east of the compass at a distance equal to at least ten times the length of the needle, and four times that of the magnet. The latter is pointed east and west, with the horizontal arm of the magnetometer as a sight, then reversed. In each case the deflection is determined by reading both pointers attached to the needle, the friction being relieved by slightly jarring the compass. The experiment is repeated by placing the magnet on a second pivot at an equal distance to the west of the needle, and from the mean of the four double readings thus obtained, the angle of deflection of the compass is determined, the natural tangent of which shows the ratio between the respective forces exerted upon the needle by the magnet and by the earth.

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A table which should accompany each magnet* shows for a certain distance between the pivots what may be called the *deflecting power* of a magnet, that is, the ratio of the magnetic moment, M, to the horizontal intensity, H.

In order to use the table, the distance between the pivots must be made accurately equal to that indicated for the given magnet, but the compass-needle need not be absolutely in the centre.

---

* These tables like that, for example, below, are calculated by the formula,

$$\frac{M}{H} = \tfrac{1}{2} \frac{(R^2 - r^2)^2}{R} \text{ tangent } A,$$

where $2R$ is the distance between the pivots and $2r$ that between the poles, which is assumed to be seven-tenths the length of the magnet. The proof will be found under the Mirror Magnetometer, not required in 1884-85.

TABLE FOR THE DEFLECTING POWER OF MAGNET X.

Distance between pivots = 50 cm. east and west. Distance between poles = 4.5 cm.

Deflection.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
10°	1290	1303	1316	1329	1342	1356	1369	1382	1395	1409
11°	1422	1435	1448	1462	1475	1488	1501	1515	1528	1541
12°	1555	1568	1581	1595	1608	1622	1635	1648	1662	1675
13°	1689	1702	1716	1729	1743	1756	1770	1783	1797	1810
14°	1824	1837	1851	1864	1878	1892	1905	1919	1933	1946
15°	1960	1974	1987	2001	2015	2028	2042	2056	2070	2084
16°	2097	2111	2125	2139	2153	2167	2181	2194	2208	2222
17°	2236	2250	2264	2278	2292	2306	2320	2334	2348	2363
18°	2377	2391	2405	2419	2433	2447	2462	2476	2490	2504
19°	2519	2533	2547	2561	2576	2590	2605	2619	2633	2648

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Making sure that the time of oscillation and hence the directive force, MH, is unchanged, we may find from the table appended the value of the horizontal intensity, H, which is of importance as the basis of the absolute magnetic system of electrical measurements.

EXAMPLE.

Directive force of magnet X from magnetic pendulum, ...	49.63
Distance between pivots, outside . .	50.34 cm.
Mean diameter of pivots	0.34 cm.
Distance between centres as in table	50.00 cm.

Deflections:

Pole of Compass-needle.	Magnet East.		Magnet West.	
North.	13°.2 E.	13°.4 W.	13°.2 E.	13°.2 W.
South.	13°.4 W.	13°.2 E.	13°.2 W.	13°.2 E.

Mean	13°.25
Second series, mean	13°.05
Third " "	13°.30
Average	13°.20
Deflecting power, from table	1716
Horizontal intensity from table:	
For directive force 49	0.1690
" " " 50	0.1707
" " " 49.63, by interpolation	0.170

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Table for the Horizontal Intensity of the Earth's Magnetism. (Part I.)

Directive Force.	Deflecting Power.										
10	367	363	359	354	350	346	342	338	334	330	327
11	404	399	394	390	385	381	376	372	368	363	359
12	441	435	430	425	420	415	410	406	401	396	392
13	477	472	466	461	455	450	445	439	434	429	424
14	514	508	502	496	490	484	479	473	468	462	457
15	551	544	538	531	525	519	513	507	501	495	490
16	588	581	574	567	560	554	547	541	535	528	522
17	624	617	610	602	595	588	581	575	568	561	555
18	661	653	645	638	630	623	616	608	601	595	588
19	698	690	681	673	665	657	650	642	635	628	620
20	735	726	717	709	700	692	684	676	668	661	653
21	771	762	753	744	735	727	718	710	702	694	686
22	808	798	789	779	770	761	752	744	735	727	718
23	845	835	825	815	805	796	787	777	768	760	751
24	882	871	861	850	840	830	821	811	802	793	784
25	918	907	896	886	875	865	855	845	835	826	816
26	955	944	932	921	910	900	889	879	869	859	849
27	992	980	968	957	945	934	923	913	902	892	882
28	1028	1016	1004	992	980	969	958	946	936	925	914
29	1065	1052	1040	1027	1015	1003	992	980	969	958	947
30	1102	1089	1076	1063	1051	1038	1026	1014	1002	991	980
31	1139	1125	1112	1098	1086	1073	1060	1048	1036	1024	1012
32	1175	1161	1147	1134	1121	1108	1094	1082	1069	1057	1045
33	1212	1198	1183	1169	1156	1142	1129	1115	1103	1090	1078
34	1249	1234	1219	1205	1191	1177	1163	1149	1136	1123	1110
35	1286	1270	1255	1240	1226	1211	1197	1183	1169	1156	1143
36	1322	1306	1291	1276	1261	1246	1231	1217	1203	1189	1176
37	1359	1343	1327	1311	1296	1280	1265	1251	1236	1222	1208
38	1396	1379	1363	1346	1331	1315	1300	1284	1270	1255	1241
39	1433	1415	1398	1382	1366	1350	1334	1318	1303	1288	1273
40	1469	1452	1434	1417	1401	1384	1368	1352	1336	1321	1306
41	1506	1488	1470	1453	1436	1419	1402	1386	1370	1354	1339
42	1543	1524	1506	1488	1471	1453	1436	1420	1403	1387	1371
43	1579	1561	1542	1524	1506	1488	1471	1453	1436	1420	1404
44	1616	1597	1578	1559	1541	1523	1505	1487	1470	1453	1437
45	1653	1633	1614	1594	1576	1557	1539	1521	1503	1486	1469
46	1690	1669	1649	1630	1611	1592	1573	1555	1537	1519	1502
47	1726	1706	1685	1665	1646	1626	1607	1589	1570	1552	1535
48	1763	1742	1721	1701	1681	1661	1642	1622	1604	1585	1567
49	1800	1778	1757	1736	1716	1696	1676	1656	1637	1618	1600
50	1837	1814	1793	1772	1751	1730	1710	1690	1671	1651	1633
51	1873	1851	1829	1807	1786	1765	1744	1724	1704	1685	1665
52	1910	1887	1864	1842	1821	1799	1778	1758	1737	1718	1698
53	1947	1923	1900	1878	1856	1834	1813	1791	1771	1751	1731
54	1983	1960	1936	1913	1891	1869	1847	1825	1804	1784	1763
55	2020	1996	1972	1949	1926	1903	1881	1859	1838	1817	1796
Horizontal Intensity.	.165	.166	.167	.168	.169	.170	.171	.172	.173	.174	.175

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Table for the Horizontal Intensity of the Earth's Magnetism. (Part II.)

Directive Force.	Deflecting Power.										
55	2020	1996	1972	1949	1926	1903	1881	1859	1838	1817	1796
56	2057	2032	2008	1984	1961	1938	1915	1893	1871	1850	1829
57	2094	2069	2044	2020	1996	1972	1949	1927	1904	1883	1861
58	2130	2105	2080	2055	2031	2007	1984	1960	1938	1916	1894
59	2167	2141	2116	2090	2066	2042	2018	1994	1971	1949	1927
60	2204	2177	2151	2126	2101	2076	2052	2028	2005	1982	1959
61	2241	2214	2187	2161	2136	2111	2086	2062	2038	2015	1992
62	2277	2250	2223	2197	2171	2145	2120	2096	2072	2048	2024
63	2314	2286	2259	2232	2206	2180	2155	2129	2105	2081	2057
64	2351	2323	2295	2268	2241	2215	2189	2163	2138	2114	2090
65	2387	2359	2331	2303	2276	2249	2223	2197	2172	2147	2122
66	2424	2395	2367	2338	2311	2284	2257	2231	2205	2180	2155
67	2461	2431	2402	2374	2346	2318	2291	2265	2239	2213	2188
68	2498	2468	2438	2409	2381	2353	2326	2298	2272	2246	2220
69	2534	2504	2474	2445	2416	2388	2360	2332	2305	2279	2253
70	2571	2540	2510	2480	2451	2422	2394	2366	2339	2312	2286
71	2608	2577	2546	2516	2486	2457	2428	2400	2372	2345	2318
72	2645	2613	2582	2551	2521	2491	2462	2434	2406	2378	2351
73	2681	2649	2618	2586	2556	2526	2497	2467	2439	2411	2384
74	2718	2685	2653	2622	2591	2561	2531	2501	2472	2444	2416
75	2755	2722	2689	2657	2626	2595	2565	2535	2506	2477	2449
76	2792	2758	2725	2693	2661	2630	2599	2569	2539	2510	2482
77	2828	2794	2761	2728	2696	2664	2633	2603	2573	2543	2514
78	2865	2831	2797	2764	2731	2699	2668	2636	2606	2576	2547
79	2902	2867	2833	2799	2766	2734	2702	2670	2639	2609	2580
80	2938	2903	2869	2834	2801	2768	2736	2704	2673	2642	2612
81	2975	2939	2904	2870	2836	2803	2770	2738	2706	2675	2645
82	3012	2976	2940	2905	2871	2837	2804	2772	2740	2708	2678
83	3049	3012	2976	2941	2906	2872	2839	2805	2773	2741	2710
84	3085	3048	3012	2976	2941	2907	2873	2839	2807	2774	2743
85	3122	3085	3048	3012	2976	2941	2907	2873	2840	2808	2776
86	3159	3121	3084	3047	3011	2976	2941	2907	2873	2841	2808
87	3196	3157	3120	3082	3046	3010	2975	2941	2907	2874	2841
88	3232	3193	3155	3118	3081	3045	3010	2974	2940	2907	2873
89	3269	3230	3191	3153	3116	3080	3044	3008	2974	2940	2906
90	3306	3266	3227	3189	3151	3114	3078	3042	3007	2973	2939
91	3343	3302	3263	3224	3186	3149	3112	3076	3040	3006	2971
92	3379	3339	3299	3260	3221	3183	3146	3110	3074	3039	3004
93	3416	3375	3335	3295	3256	3218	3181	3143	3107	3072	3037
94	3453	3411	2371	3330	3291	3253	3215	3177	3141	3105	3069
95	3489	3448	3406	3366	3326	3287	3249	3211	3174	3138	3102
96	3526	3484	3442	3401	3361	3322	3283	3245	3207	3171	3135
97	3563	3520	3478	3437	3396	3356	3317	3279	3241	3204	3167
98	3600	3556	3514	3472	3431	3391	3352	3312	3274	3237	3200
99	3636	3593	3550	3508	3466	3426	3386	3346	3308	3270	3232
100	3673	3629	3586	3543	3501	3460	3420	3380	3341	3303	3265
Horizontal Intensity.	.165	.166	.167	.168	.169	.170	.171	.172	.173	.174	.175

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MIRROR MAGNETOMETER.

[Reserved in 1884-85.]

The Determination of Magnetic Moment and Horizontal Intensity by the Magnetic Pendulum and Mirror Magnetometer.

The experiment with the magnetic pendulum is repeated, but instead of observing the turning points, the times are noted when the pendulum passes the magnetic meridian. A telescope and scale are employed for this purpose, being so adjusted that when the magnet is at rest and pointing horizontally north and south, a cross-hair coincides with the zero of the scale reflected by the polished end of the magnet.

A small oscillation, not exceeding an arc of 1° in amplitude, is then communicated to the magnet, and its position is determined at every tick of the seconds clock just before and just after the passing of the zero, the exact time of which is afterwards found by interpolation within a tenth of a second, so that the average time of oscillation, t, may be reduced very exactly by the method of consecutive observations.

The time, T, of oscillation under torsion is also roughly determined by causing the magnet to oscillate through several whole revolutions, and as directive forces are to each other inversely as

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the squares of the times of oscillation, we have*

$$MH = D = \frac{\pi^2 K}{t^2}\left(1 - \frac{t^2}{T^2}\right).$$

The magnet is then mounted upon pivots as in the ordinary magnetometer, but at considerably greater distances, and the deflections of a small magnet, carrying the necessary mirror and suspended by a fine thread, are observed by means of a telescope and scale.

At any two distances, R_1 and R_2, let the deflections be A_1 and A_2, and let a be the small deflection in degrees produced by twisting the thread through one whole revolution, then we have nearly,

$$\frac{M}{H} = \frac{R_1^5\, tangent\, A_1 - R_2^5\, tangent\, A_2}{2\,(R_1^2 - R_2^2)}\left(1 + \frac{a}{360}\right).$$

The formula is established as follows: let r be the distance of the centre of the magnet from either pole, whose strength is $\pm m$, and let R be the distance of the compass needle, which must be several times shorter than the magnet, in order that its length may be disregarded; the field of force due to the nearer pole is then $\pm \frac{m}{(R-r)^2}$, and that due to the other is $\mp \frac{m}{(R+r)^2}$.

* For the proof, see under Kater's Pendulum.

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The resultant field, F, being the algebraic sum of these two quantities, we have,

$$F = \pm \frac{m}{(R-r)^2} \mp \frac{m}{(R+r)^2}$$

$$= \pm \frac{4mrR}{(R^2-r^2)^2}, \text{ or since } M = 2mr,$$

$$F = \pm \frac{2MR}{(R^2-r^2)^2}.$$

But from the triangle of forces, we have

$$F = \pm H \times \textit{tangent } A,$$

$$\text{hence } \frac{M}{H} = \tfrac{1}{2}\frac{(R^2-r^2)^2}{R} \times \textit{tangent } A.*$$

---

* Experiment shows that $r^2 = \frac{1}{8}L^2$ very nearly, where L is the length of the magnet, and since $R = \frac{1}{2}D$, we have

$$\frac{M}{H} = \tfrac{1}{16}D^3\left(1 - \frac{L^2}{D^2} + \tfrac{1}{4}\frac{L^4}{D^4}\right) \textit{tangent } A, \text{ nearly,}$$

from which, disregarding the last term in the parenthesis, we have

$$\frac{M}{H} = \tfrac{1}{16}\, D \cdot (D+L) \cdot (D-L) \cdot \textit{tangent } A, \text{ nearly,}$$

by which the deflecting power may be approximately calculated.

The error due to the position of the poles being inversely as the square of the successive distances, $D_1 D_2 \text{ - - - - } \infty$, between the pivots in different experiments, the corresponding values, $V_1 V_2 \text{ - - - - } V\infty$, will be connected by the equation,

$$\frac{V\infty - V_1}{V_1 - V_2} = \frac{\left(\frac{1}{\infty}\right)^2 - \left(\frac{1}{D_1}\right)^2}{\left(\frac{1}{D_1}\right)^2 - \left(\frac{1}{D_2}\right)^2},$$

hence the true value may be found by the formula,

$$V\infty = V_1 + (V_2 - V_1)\frac{(D_2)^2}{(D_2)^2 - (D_1)^2}.$$

The correction is justified only when $V_2 - V_1$ is considerable beside the errors of observation. The approximate formula, accordingly, is often in practice to be preferred.

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By obvious transformations we have

$$R_1^5 \text{ tangent } A_1 = \frac{2MR_1^2}{H} \div \left(1 - \frac{2r^2}{R_1^2} + \frac{r^4}{R_1^4}\right)$$

$$= \frac{2MR_1^2}{H} + \frac{4Mr^2}{H} \text{ nearly,}$$

also

$$R_2^5 \text{ tangent } A_2 = \frac{2MR_2^2}{H} + \frac{4Mr^2}{H} \text{ , nearly,}$$

whence subtracting,

$$R_2^5 \text{ tangent } A_2 - R_1^5 \text{ tangent } A_1$$

$$= \frac{2MR_2^2}{H} - \frac{2MR_1^2}{H} \text{ ,}$$

from which the formula above directly follows, if we remember that the magnet has to overcome the directive force of torsion as well as that of the earth's magnetism, the ratio between these directive forces being expressed by the quotient of the two angles for which the forces are in equilibrium.

Measurements of angles less than one degree are reduced with sufficient accuracy for four or five significant figures by dividing the deflection in centimetres by twice the distance of the mirror from the scale.

The magnetic moment and horizontal intensity are finally calculated by the formulæ,

$$M = \sqrt{MH \times \frac{M}{H}} \text{ and } H = \sqrt{MH \div \frac{M}{H}}.$$

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DISTRIBUTION OF MAGNETISM.

[Not required in 1884-85.]

The Investigation of the Distribution of Magnetism upon a Bar by means of a Sliding Helix.

A piece of brass tube is slipped over the bar and clamped in different places. A coil of wire is suddenly made to slide over the brass tube through a certain distance determined by two stops, noting the throw of the needle of a galvanometer connected with the terminals of the coil.

The mean distances of the stops from the centre of the bar are represented graphically by points along a horizontal line, and the corresponding deflections by perpendiculars at those points, the extremes of which are connected by a curved line, representing the distribution of magnetism upon the bar.

The centres of gravity of the two areas enclosed may be marked by the eye,* and indicate the positions of the magnetic poles, the distance between which is therefore roughly determined.

* A more accurate method would be to cut out the areas from a heavy sheet of metal or card-board, and suspending each at two points, to mark the direction of the plumb-lines, which intersect at the centre of gravity.

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MAGNETIC DIP.

The Determination of the Magnetic Dip by means of the Earth-Inductor.

The large coil of the earth-inductor, containing 72 turns, is rotated rapidly first about a vertical, then about a horizontal axis,* in each case as nearly as possible through two right angles. In the first case, the plane of the coils should be vertical east and west, in the second, horizontal, before and after turning.

The ratio of the small throws† of the needle of a galvanometer, connected with the terminals of the coil, gives the ratio between the horizontal and the vertical intensity of the earth's magnetism, that is, the cotangent of the angle of the dip, which is thus easily found from the mean of several determinations.

* This is most accurately accomplished by means of a spring set in motion by pulling a detent, and arrested at the proper time by a suitable catch and buffer.

† For greater strictness, the ratio of the sines of half the angles of throw should be employed.

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EXAMPLE.

Throw of needle.

Axis vertical.	Axis horizontal.
4°.5	15°.8
4°.4	16°.0
4°.6	16°.2
5°.0	16°.3
4°.8	15°.7
4°.9	16°.3
4°.1	15°.9
4°.3	16°.5
4°.7	15°.5
5°.0	15°.8
Average 4°.63	16°.00

Quotient (4.63 ÷ 16.) 0.289

Angle whose cotangent is .289

= angle of dip . . . . . 73°.9

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REDUCTION FACTOR.

The Determination of the Constant and Reduction Factor of a Galvanometer.

I. *Direct Method.* The constant of a galvanometer is equal to the field of magnetic force produced at the centre of the needle by a unit of current, which by definition creates one unit of field at unit distance for every unit length of wire. Since the action diminishes as the square of the distance increases, the constant, X, of a single-ring galvanometer must be equal to the total length of wire divided by the mean square of its distance from the needle.

The necessary data are obtained by obvious measurements. The outside diameter or circumference of the coil and the number of turns may, for instance, be ascertained, together with the semi-diameter of the wire, including the insulating material, which must be taken into account in finding the distance of the middle of the wire from the centre of the coil.

The ring is then adjusted in the plane of the magnetic meridian, so that the two ends of the needle point as nearly as possible to 0° and 180°, respectively, under the influence of the earth's magnetism.

The field of force produced by any current, C,

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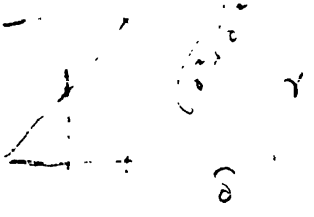

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deflecting the needle of the galvanometer through an angle, A, against the horizontal intensity, H, of the earth's magnetism, is of course CX; hence from the triangle of forces,

$$CX : H :: \textit{tangent } A : 1, \text{ that is}$$

$$C = \frac{H}{X} \textit{ tangent } A.$$

This quantity, $\frac{H}{X}$, by which the tangent of the deflection must be multiplied, in order to find the current, is called the reduction factor of the galvanometer.

II. *Method of Comparison.* A second galvanometer is adjusted in the same way as the first, and a current is sent in series through both, being sufficiently strong to produce measurable deflections; and these should be confirmed by reversing the current.

Care must be taken to neutralize the effect of the two connecting wires in each case by twisting them together, so as to form a parallel system carrying opposite currents, and the two galvanometers should be separated by a considerable distance in order to avoid mutual action.

Evidently the constant of the second galvanometer is to that of the first directly, and its reduction factor inversely as the tangents of the angles of deflection produced by a given current.

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OSCILLATION OF NEEDLE.

The Comparison of Two Galvanometers by the Method of Oscillation.

The constant of a double-ring galvanometer is calculated by the formula

$$X = \frac{2\pi r^2 n}{(r^2 + d^2)^{\frac{3}{2}}},$$

where n is the number of turns of radius, r, and d is half the distance between the two rings, one on each side of the compass. For the total length of wire is $2\pi rn$, which must be divided by the square of the distance from the needle, $r^2 + d^2$, and multiplied by the cosine of the angle between the forces and their common resultant, in every case equal to $r \div \sqrt{r^2 + d^2}$, to find the constant in question. The horizontal intensity of the earth's magnetism divided by this constant, gives the reduction factor of the double-ring galvanometer, which should agree closely with that obtained by the method of comparison.

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The radius is found from the mean of obvious measurements of the outside and inside diameter of the coils, and d is half the mean of the outside and inside distances between the rings.

An electric current is passed through the galvanometer whose constant has thus been determined, sufficient to produce a measurable deflection, thence through a second galvanometer, taking care to twist together, in each case, the terminals of insulated wire. In the ordinary method of adjustment, the two constants are directly as the tangents of the corresponding deflections, that is,

$$\frac{X}{X'} = \frac{tangent\ A}{tangent\ A'}.$$

When, however, the second galvanometer is too sensitive to give a measurable deflection, we may rotate the instrument through 90° and notice the time, t, of oscillation under the influence of the current. If t_0 is the ordinary time of oscillation, the tangent of the angle of deflection in the formula may be replaced by the expression,

$$\frac{t_0^2}{t^2} \pm 1.$$

For if F is the field of force due to the current and H that due to the earth, this tangent corre-

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sponds to the quotient, $F \div H$; and since the strength of a field is proportional to the square of the frequency of oscillation,

$$F \mp H : H :: t_o^2 : t^2, \text{ whence we find}$$

$$\frac{F}{H} = \frac{t_o^2}{t^2} \pm 1^*.$$

The sign is to be read *plus* if the current reverses or tends to reverse the needle; otherwise *minus*.

The natural time of oscillation should be found before and after the experiment,† since it is likely to be altered by strong currents, to which it must be remembered that, owing to charges of induced magnetism, the formula does not strictly apply.

---

* In the case of a needle suspended by a thread and oscillating under torsion in the time, T, we should strictly multiply the right-hand member by the factor $1 + \frac{t^2}{T^2}$, since the directive force of the thread is to that of the earth's magnetism inversely as the squares of the times of oscillation.

† For an astatic needle, the "constant" is continually changing, owing to variation in the distribution of magnetism, hence also the reduction factor, which is, moreover, influenced by changes in the horizontal intensity; but having found in any one case the reduction factor and the time of oscillation, the former may be calculated in any other case by multiplying into the square of the ratio of the old time of oscillation to the new.

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EXAMPLE.

Diameter of rings, outside, 10 obs. .	33.06 cm.
Radius " " "	16.53 cm.
Depth to outer surface of wire . .	1.02 cm.
Outer radius of coil	15.51 cm.
Width of five coils of wire on each ring	1.2 cm.
Semi-diameter	0.12 cm.
Mean radius of coil (15.51 — 0.12) .	15.39 cm.
Outer distance between rings . . .	16.68 cm.
Mean width of rings	2.16 cm.
Half mean distance, $\frac{1}{2}(16.68 - 2.16)$	7.26 cm.
Constant, $\frac{2 \times 3.1416 \times 15.39^2 \times 10}{(15.39^2 + 7.26^2)^{\frac{3}{2}}} =$	3.020
Reduction factor, .1703 ÷ 3.02 =	0.0564
II. Deflection of tangent galvanometer	30°.88
Tangent of 30°.88	0.598
Time of oscillation of astatic galvanometer reversed by current in double arc .	0.198 sec.
Do. without current after experiment	3.93 sec.
Ratio of fields of force, $\frac{3.93^2}{.198^2} + 1 =$ . .	395
Constant, $3.02 \times \frac{395}{.598} =$	1995

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GALVANOMETER AND SHUNT.

[Omitted in 1884–85.]

The Comparison of the Constant and Reduction Factor of Two Galvanometers by means of a Shunt.

When a very sensitive galvanometer is to be compared with one of the ordinary sort, the deflection of the former may be conveniently reduced by means of a shunt, so that the same current causes measurable deflections in both cases. The reduction factors and constants may then be compared in the usual way; but it must be remembered that, without the shunt, the constant will be greater and the reduction factor less in the known ratio by which the current has here been reduced.

This ratio is generally marked upon each of the shunts which accompany a delicate instrument, and may be confirmed, later on, by means of Wheatstone's Bridge. It will be found that if S be the resistance of the shunt and G that of the galvanometer, the current will be reduced, by Ohm's Law, in the ratio which $S + G$ bears to S.

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ELECTRO-CHEMICAL METHOD.

The Determination of the Reduction Factor of a Galvanometer by the Electro-Chemical Method.

The ordinary positive pole of a Daniell's cell is replaced by a weighed spiral of copper wire, which is re-weighed after allowing the current to flow for a given time, t, through the spiral and the coils of a galvanometer, noting at regular intervals the deflection, which is most accurately determined in the neighborhood of 45°. If m is the mass of copper deposited, the current, C, is given in absolute measure by the formula

$$C = 299.4\frac{m}{t}\text{, nearly,}$$

since the unit of current deposits one gramme in about 299.4 seconds, and if *tangent A* is the mean tangent* of the angle of deflection during the experiment, the reduction factor is evidently

$$\frac{H}{X} = \frac{C}{\textit{tangent } A} = \frac{299.4\ m}{t\ \textit{tangent } A}\ .$$

* When the deflection is constant within one or two per cent. and not greater than 45°, the tangent of the mean angle of deflection may be used instead of the mean tangent of the angle.

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The copper spiral carefully cleaned with nitric acid, is immersed in a solution saturated with sulphate of copper free from all impurities, especially acid, ammoniacal, and oxidizing or reducing agents. It should be freshly coated with copper by the action of the battery at the beginning of the experiment, and before weighing it should in all cases be removed from the bath, with care not to dislodge any of the deposit, dipped into water, then alcohol, and thoroughly dried at a temperature not exceeding 100°, to avoid oxidation of the copper.

In repeating the experiment, the direction of the current should be reversed, to eliminate the zero error due to the imperfect adjustment of the galvanometer, and care should be taken in both cases to neutralize the effects of the connecting wires, by twisting them together.

The result is to be compared with the reduction factor already determined.

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EXAMPLE.

Weight of copper spiral, mean of 3 double weighings	18.437 g.
Ditto after experiment	19.000 g.
Weight of copper deposited . . .	0.563 g.
Duration of experiment, 50 minutes less 6 seconds	2994 sec.
Mean deflection taken every minute,	
1st 10 minutes . .	46°.29
2d " " . .	45°.60
3d " " . .	44°.73
4th " " . .	44°.47
5th " " . .	43°.91
Average	45°.00
Tangent of 45°	1.000
Reduction factor, $\frac{299.4 \times .563}{2994 \times 1} =$	0.0563
Compare with result of direct measurement	0.0564

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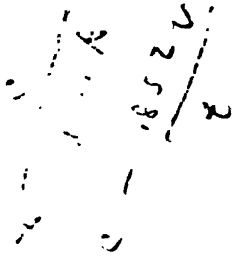

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GAS VOLTAMETER.

[Not required in 1884-85.]

The Determination of the Reduction Factor of a Galvanometer by means of a Gas Voltameter.

A battery of four or five Daniell's cells is connected through a tangent galvanometer with a small voltameter containing dilute sulphuric acid; if v is the volume of dry hydrogen gas at 0° and 76 cm. liberated in the time, t, and if *tangent A* is the mean tangent of the angle of deflection, then the reduction factor is given by the equation,

$$\frac{H}{X} = \frac{0.852\, v}{t \text{ tangent } A}, \text{ nearly,}$$

since the unit of current sets free one cubic centimetre of hydrogen in about 852 thousandths of a second.*

The polarization of the electrodes causes at first a rapid diminution of the current; but observations of the deflection and volume liberated may be begun at regular intervals as soon as these quantities are sufficiently constant.

In repeating the experiment, the galvanometer terminals should be interchanged. The mean of the two results is compared with that obtained by the deposition of copper.

* Assuming that dry hydrogen at 0° and 76 cm. has a density, 0.00008962, and that the unit of current sets free one gramme of hydrogen in 9505 seconds.

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Example.

I. Volume of hydrogen at beginning of experiment . . 1.50 cu.cm.
Volume of hydrogen at end . . 4.00 cu.cm.

Volume liberated at 17° . . . . 2.50 cu.cm.
Pressure of barometer . . . . . 76.44 cm.
Tension of aqueous vapor at 17° . . 1.44 cm.

Pressure of dry hydrogen . . . . 75.00 cm.
Volume reduced to 0° and 76 cm.

$$2.50 \times \frac{75}{76} \times \frac{273}{290} = \quad . \quad . \quad . \quad 2.32 \text{ cu.cm.}$$

Time required . . . . . 520 sec.
Mean deflection, 4°.4 to 3°.6, N. W. & S. E. . 3°.92
Tangent of 3°.92 . . . . . 0.0685

Reduction factor, $\frac{0.852 \times 2.32}{520 \times .0685} =$ 0.0555

II. Corrected volume, same way . 2.32 cu.cm.
Time required . . . . . 580 sec.
Mean deflection, 6°.0 to 2.°2, N. E. & S. W. . 3°.44
Tangent of 3°.44 . . . . . 0.0600

Reduction factor, $\frac{0.852 \times 2.32}{580 \times .06} =$ 0.0568

Mean of I. and II. 0.0562
Compare with result from deposition of copper 0.0563

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ELECTRO-CHEMICAL EQUIVALENTS.

[Reserved in 1884-85.]

The Determination of Electro-Chemical Equivalents.

A series of batteries and voltameters is arranged so that the same current passes through each. The relative amounts of different elements dissolved, precipitated, or set free at the various poles or electrodes, are determined as in the case of copper or hydrogen, by weight or by volume. These are called electro-chemical equivalents, and are definite only when the salt formed or broken up has a definite chemical composition. A thorough study and understanding of the conditions and nature of the reactions is therefore requisite for the simplest original investigation.

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RESISTANCE BY HEATING.

The Determination of the Absolute Electrical Resistance of a Conductor by means of a Calorimeter.

A current, measured by a tangent galvanometer whose reduction factor is known, is passed through a coil of wire surrounded with a weighed quantity of distilled water, and the rise of temperature accurately determined.

The wire, which may be naked, is carefully bent so as not to touch the sides of the calorimeter nor itself; it is joined at both ends to terminals of thick copper wire, somewhat below the surface of the water, which is briskly agitated during the experiment to secure uniformity of temperature.

The times are noted when the mercury passes each degree or half degree upon the stem of the thermometer, and between the observations the deflection of the galvanometer, which should be nearly constant, is repeatedly determined.

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The quantity of heat, $\mathcal{Q}$, developed in the time, t, is then calculated in the ordinary way, or better, if possible, by the method of consecutive observations; then multiplying by $\mathcal{J}$, the mechanical equivalent of heat, say 41,660,000 ergs per gramme-degree, we find the corresponding amount of work, $\mathcal{J}\mathcal{Q}$.

The electrical resistance, R, is defined as the work done by the unit of current in passing through the wire for the unit of time. The opposing force increases as in a capillary tube in proportion to the current, C, while the quantity of electricity which overcomes this force increases as the current, C, as well as in proportion to the time, t. The work done is given accordingly, by Joule's Law,

$$W = \mathcal{J}\mathcal{Q} = C^2Rt.$$

If, therefore, A is the mean deflection of the galvanometer, and $\frac{H}{X}$ its reduction factor, the electrical resistance of the wire is determined by the formula,

$$R = \frac{\mathcal{J}\mathcal{Q}}{t(\frac{H}{X}\,tangent\ A)^2}.$$

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Example.

Weight of water in calorimeter . . .	68.3 g.
Thermal capacity of calorimeter, etc. .	4.7 g.
Total	73.0 g.
Time considered, 14 m. 18 s. = . . .	858 sec.
Rise of temperature	3°.50
Average excess over temp. of room . . .	2°.50
Cooling for 1 sec. and 1° excess* . . .	0°.0002
Correction for cooling, $2.50 \times 858 \times .0002$.	0°.429
Heat generated, $73.0 \times (3°.50 + 0°.429) =$	286.8
Mean deflection of galvanometer . . .	46°.7
Tangent of 46°.7	1.0612
Reduction factor for double arc (half current through each coil) $2 \times .0564 =$	0.1128
Resistance of wire, $\frac{41{,}660{,}000 \times 286.8}{858 \times .1128^2 \times 1.0612^2} =$ . .	0.972×10^9

* Determined by special investigation. See note and example under Specific Heat of Solids.

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WHEATSTONE'S BRIDGE.

The Comparison of Resistances by means of the British Association Form of Wheatstone's Bridge.

The poles of a small battery are connected with the two ends of a straight wire of considerable resistance. A portion of the current flows through this wire; another portion takes a parallel course through a thick strip of copper, interrupted in two places by a known resistance, r, and an unknown resistance, x. Between r and x, one of the terminals of a delicate galvanometer is attached, the other terminal being carried by means of a sliding contact-piece to a point of the same potential on the straight wire, making a cross-connection, called a bridge, between the two parallel circuits.

It may be observed that electric potential corresponds to hydrostatic pressure, or head of water, and that difference of potential, like loss of head, is measured by the work done by the unit quantity of electricity in passing from one point to another. Since also resistance is proportional, *ceteris paribus*, to the work done by the current, the fall of potential throughout any simple con-

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ductor must be proportional to the resistance which the current has had to overcome.

The potential at any point of either of two parallel circuits may therefore be calculated from that of the two junctions by simple interpolation in the ratio of the resistances which lie on either side of that point; hence if the potential be the same at two points, one on each wire, the ratios of these resistances must be equal in the two cases.

The two extremities of the bridge are known to have the same potential when, on alternately making and breaking the battery circuit, the needle, though free to swing, remains at rest. The distances a and b of the point of contact from the two ends of the straight wire nearest, respectively, to r and x, are then read off by means of the graduated scale, and the unknown resistance, x, is calculated from the proportion,

$$a : b :: r : x.$$

It is of interest to compare a copy of the standard ohm or British Association unit with the coil whose resistance has been tested by heating in water. Allowance must of course be made for the resistance of the connecting wires,* and the

* The resistance of 68 centimetres of commercial copper wire, of 0.13 cm. diameter, No. 18, B. W. G. may be taken as 10 million absolute units.

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error due to the inequality of the two ends of the straight wire should be eliminated in so far as possible by interchanging the places of the known and unknown resistances.

EXAMPLE.

Resistance measured by heating .	0.972×10^9
Resistance of 41 cm. No. 18 wire, B. W. G. at $.010 \times 10^9$ for 68 cm.	0.006×10^9
Total	0.978×10^9
Distances from ends	50.41 / 49.59 } cm.
Ratio, $50.41 \div 49.59 =$	1.0165
Distances after interchange . . .	50.42 / 49.58 } cm.
Ratio, $50.42 \div 49.58 =$	1.0169
Mean ratio	1.0167
Unknown resistance marked 1 ohm $1.0167 \times 0.978 \times 10^9 =$ . . .	0.994×10^9
Second specimen, same way . .	0.990×10^9
Mean	0.992×10^9
Compare with rough determination of standard, Cambridge, 1885 .	0.987×10^9
1 B. A. unit acc. to Int. Congress, Paris, 1884 = ...	0.9889×10^9
Ditto acc. to F. Kohlrausch . .	1.0196×10^9
Ditto acc. to Lorenz	0.9797×10^9
Ditto acc. to H. F. Weber . . .	1.0165×10^9
Intended value	1.000×10^9

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BOX OF COILS.

[Omitted in 1884-85.]

The Comparison of Resistances by means of a Box of Coils.

By the removal of the proper plugs, a resistance of 1000, 100, 10, 1, or $\frac{1}{10}$ ohms may be introduced into either of the fixed arms, *a* and *b*, of the inner horse-shoe. The outer horse-shoe supplies the third known resistance, *r*, which is here variable at will; and the three are joined with the unknown resistance, *x*, in the endless series, *abxra*.

The battery is usually connected with the junctions, *bx* and *ra*, while the galvanometer terminals unite *ab* and *xr*, thus completing a Wheatstone's Bridge. The resistance, *r*, is then adjusted until the galvanometer is not affected by making or breaking the battery circuit. This condition is not, in general, fulfilled by any combination of the rheostat; but by measuring the sensitiveness of the galvanometer to a change of one ohm, the fraction which would bring about

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the exact adjustment may be estimated from the deflection in any case to tenths or even hundredths of an ohm. The unknown resistance is then calculated by the usual formula,

$$x = br \div a.$$

It is sometimes of advantage to interchange the battery and galvanometer connections; and the choice of the proper resistances for the fixed arms is often of great importance as concerns the sensitiveness of the galvanometer. It is evident that a, b, r, and x should be as nearly equal as circumstances will allow, and not entirely disproportionate to the resistance of the galvanometer.

These problems may always be submitted to calculation, but are often more readily solved by the actual experiment.

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CORRECTION OF A SET OF COILS.

[Reserved in 1884–85.]

The Determination of the Resistances of a Set of Coils in Absolute Measure.

The one-ohm coils are separately compared with an accurate copy of the British Association unit, or standard ohm, by means of a Wheatstone's Bridge, finely graduated. The method of interchanging resistances is employed as heretofore whenever the greatest accuracy is required; but for rapid work, the middle point of the wire may once for all be determined, and the ratio of the known to the unknown resistance calculated from a single setting.

Assuming that the value of the standard ohm is 989 million absolute units, that of each one-ohm coil may be expressed in the same measure, and by combining one of them with the standard, in series, the resistance of the two-ohm coils may be determined. Two of these, together with the standard, are then balanced against a five-ohm coil, and thus in succession all the resistances are referred as directly as possible to the standard.

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The contact-piece will have to be moved through a small distance on either side of the middle point, and it is important to compare the linear resistance of this portion with the average value throughout the wire. For this reason, resistances are also compared which should be to each other in the ratios 1000: 1001, 1000: 1002, etc., and the agreement of the results with those previously obtained practically establishes the uniformity of the wire.

A movable screw-cup is provided for connecting the galvanometer with the brass strip joining any two resistances in the series. This therefore replaces the central copper strip of the Wheatstone's apparatus; otherwise the connections are made precisely as before.

NOTE. The resistances of various one-ohm coils were determined in 1884-85 as follows:

A	1	1.0009	× 989 millions.	C	1	1.0021	× 989 millions.
	2	1.0009	" "		2	1.0021	" "
	3	1.0005	" "		3	1.0055	" "
	4	1.0013	" "		4	1.0055	" "
B	1	1.0094 ?	" "	D	1	1.0007	" "
	2	1.0119 ?	" "	E	1	1.0015	" "
	3	1.0015	" "	F	1	0.9999	" "
	4	1.0019	" "	G	1	1.0123	" "

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SPECIFIC RESISTANCE.

The Determination of Specific Electrical Resistance.

The resistance, x, of a piece of uninsulated wire of length, l, and diameter, d, is determined in the usual method with the British Association form of Wheatstone's Bridge, using for comparison the small rheostat whose resistance has been found by heating water, or one with which it has been compared.

The specific resistance, R, is then given by the equation,

$$R = \tfrac{1}{4}\frac{\pi d^2 x}{l}\ ; —$$

For, since R is by definition the resistance to electrical conduction between opposite faces of a unit cube of the given material, and since resistance increases with the length of a conductor and is inversely as its cross-section, which is in this case $\frac{1}{4}\pi d^2$, we have

$$x = R \times l \div \tfrac{1}{4}\pi d^2, \text{ therefore etc.}$$

The length is measured by a vernier gauge between the marks made by the sharp edges of the clamps which hold the wire; the diameter is found by a micrometer in the usual way.

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EXAMPLE.

I. Length of German silver wire between clamps . . . . . 10.06 cm.

Diameter, mean of 25 obs. . . . 0.01819 cm.

Known resistance . . . . . 0.978×10^9

Ratio of distances, unknown resistance at left-hand, $46.95 \div 53.05 =$ . . 0.885

Ditto right-hand, $47.05 \div 52.95 =$ . . 0.889

Mean 0.887

Resistance, $.978 \times .887 \times 10^9$ . . 0.867×10^9

Specific resistance,

$$\frac{10^9 \times .867 \times 3.1416 \times .01819^2}{4 \times 10.06} = 22.4 \times 10^3$$

Compare with Everett's Units, etc. . 21.17×10^3

II. (*Extra.*) Length of No. 18 commercial copper wire, B.W.G. . . 6800 cm.

Diameter . . . . . 0.1300 cm.

Resistance, 1 ohm = ... 0.989×10^9

Specific resistance,

$$\frac{10^9 \times .989 \times 3.1416 \times .13^2}{4 \times 6800} = 1.93 \times 10^3$$

Compare value tabulated for pure copper . . . . . 1.65×10^3

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THOMSON'S METHOD.

[Not required in 1884-85.]

The Determination of the Resistance of a Galvanometer by Thomson's Method.

The galvanometer takes the place of the unknown resistance in a Wheatstone's Bridge apparatus, using the British Association form for a low, and a box of coils for a high resistance. It is not necessary to employ a second galvanometer to detect the existence of a current in the bridge, since part of this current must flow through the galvanometer whose resistance is being tested, tending, accordingly, to increase or diminish the deflection; and especially if the force of the current be neutralized with a permanent magnet, suitably placed, the needle will respond to the opening or closing of the bridge. When this no longer takes place, the current in the bridge is known to be zero, and the resistance of the galvanometer is to the known resistance as the corresponding segments of the straight wire in the British Association apparatus, or their equivalents in the fixed arms of a box of coils.

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MANCE'S METHOD.

[Not required in 1884-85.]

The Determination of the Internal Resistance of a Battery by Mance's Method.

The battery is put in the place of the unknown resistance in a Wheatstone's Bridge, and the deflection of the galvanometer, reduced, if necessary, with a shunt, is nearly neutralized by means of permanent magnets, suitably placed. A second battery is not necessary in this arrangement, but may be replaced by a simple key with the same connecting wires, through which a portion of the current will flow when the circuit is closed. This must be allowed to take place, in testing the adjustment of the resistances, only for a moment in each case; since otherwise the electromotive force of the battery would be affected, as is proved by the slow change in the galvanometer deflection accompanying a long depression of the key. When, however, upon closing the circuit, the galvanometer shows no *sudden* * movement

* Glazebrook and Shaw, *Practical Physics*, page 450.

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either to right or left, the bridge is known to be adjusted, and the battery resistance may be calculated by the ordinary formula for a Wheatstone's Bridge.

The method may be modified, in the case of a battery consisting of an even number of cells, by opposing them one to another, so that the galvanometer shows very little permanent deflection, and requires, accordingly, little or no adjustment. In this case, an auxiliary battery will be necessary as in Wheatstone's Bridge; and with the same apparatus, the resistance of electrolytes may be determined.

For greater exactness, alternating currents can be employed, using a double commutator acting simultaneously in the galvanometer and in the auxiliary battery circuits; but in most cases, paying attention to the instantaneous, and not to the slow changes of the galvanometer, sufficient exactness is much more simply obtained with continuous currents.

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EXAMPLE.

Length of nearer arm of wire . .	49.0 cm.
" " farther " " " . .	51.0 cm.
Ratio	0.96
Known resistance employed . .	2 ohms
Corrected value, allowing for connecting wires	2.022 ohms

Absolute value,

$$2.022 \times 0.989 \times 10^9 = 2.000 \times 10^9$$

Resistance of 2 cells, $.96 \times 2 \times 10^9 = 1.92 \times 10^9$

Average resistance of 1 cell, . 0.96×10^9

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OHM'S METHOD.

The Simultaneous Determination of the Internal Resistance and Electromotive Force of a Battery.

A small Daniell's cell is connected with a tangent galvanometer of low resistance, but with a sufficient number of turns of wire to give a measurable deflection, lying preferably between 40° and 70°. When the current is reversed, if the coils are parallel to the magnetic meridian, the needle will be turned in the opposite direction through the same angle, A_1. If the difference in the two cases is slight, the mean deflection may be taken; otherwise the instrument should be readjusted.

Sufficient resistance, r, is now added to reduce the deflection to an angle, A_2, whose tangent is about half that of A_1, lying accordingly between 20° and 50°.

Since the electromotive force, E, of the Daniell's cell is practically unaffected by the strength of the current, we find, by Ohm's Law,* that the

* This follows directly from Joule's Law, $W = C^2Rt$, introduced in the determination of resistance by heating; for E is the work done by the unit of electricity in overcoming the resistance, R, under the given conditions, and is evidently equal to the total work, W, divided by the quantity of electricity, Ct, that is, $E = \frac{C^2Rt}{Ct} = CR$, whence $C = \frac{E}{R}$.

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current, and hence also the tangent of the deflection, must be inversely as the total resistance of the circuit, that is

$$x : x + r :: \textit{tangent } A_2 : \textit{tangent } A_1, \text{ whence}$$

$$x = \frac{r \textit{ tangent } A_2}{\textit{tangent } A_1 - \textit{tangent } A_2},$$

from which the internal resistance of the battery may be calculated; but it should be remembered that the resistance of the galvanometer and that of the connecting wires are also strictly included in the quantity, x.

By Ohm's Law, again, we find the electromotive force, which is equal to the product of the resistance, x, by the current, $\frac{H}{X}$ *tangent* A_1.*

Care must be taken as usual in the arrangement of the connecting wires, and the necessary resistance may be introduced by means of a set of coils provided for the purpose, using if possible those whose absolute values have been already determined.

---

* E may also be calculated directly by the formula,

$$E = \frac{rH}{X} \cdot \frac{\textit{tangent } A_1 \textit{ tangent } A_2}{\textit{tangent } A_1 - \textit{tangent } A_2}.$$

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EXAMPLE.

I.	Resistance of 1-ohm coil No. 1	0.994×10^9
	" " " " No. 2	0.990×10^9
	Total resistance introduced . . .	1.984×10^9
	Reduction factor of galvanometer . . .	0.0564
	Deflection, mean of 3 readings . . .	63°.5
	Ditto with current reversed	63°.1
	Mean of 6 readings	63°.3
	Ditto with additional resistance	34°.9
	Tangent of 63°.3	1.988
	" " 34°.9	0.698
	Internal resistance,	
	$\frac{10^9 \times 1.984 \times .698}{1.988 - .698} =$ . . .	1.07×10^9
	Electromotive force of Daniell's cell,	
	$10^9 \times 1.07 \times 1.988 \times .0564 =$	1.20×10^8
II.	Second Daniell's cell, same way,	
	Resistance	0.989×10^9
	Electromotive force . . .	1.16×10^8

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BEETZ' METHOD.

The Determination of Battery Resistance and the Comparison of Electromotive Forces by Beetz' Method.

Two batteries which are to be compared are joined by similar poles in multiple arc to the extremities of a variable resistance, so that the stronger battery, whose electromotive force is E and internal resistance B, by means of the current, c, through the resistance, r, exactly neutralizes the electromotive force, e, of the weaker battery, as is shown by a sensitive galvanometer in the circuit of the latter.

This occurs when the difference of potential at the two extremities of r, given by Ohm's Law as cr, is equal to the weaker electromotive force, e, that is, when

$$e = cr.$$

Now reducing the current to an amount, c', by the additional resistance, R, in the circuit of the stronger battery, the deflection of the galvan-

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ometer is again brought to zero by giving to the variable resistance a value, r', therefore

$e = c'r' = cr$, whence

$c : c' :: r' : r$, but by Ohm's Law,

$c : c' :: r' + R + B : r + B$, hence

$$\frac{r'}{r} = \frac{r' + R + B}{r + B}, \text{ so that we find}$$

$$B = \frac{rR}{r' - r},$$

from which the internal resistance of the battery may be calculated.*

Care must be taken to close the circuit of the stronger battery only for a moment, since the electromotive force begins to change as soon as a current is established, and hence also the ratio of the resistances producing equilibrium in the weaker circuit. The latter should never be closed except during the short interval in question, using a double key especially constructed so as to make the stronger circuit first and break it last. Sufficient time should be allowed for the stronger battery to recuperate, especially before the final test of the adjustment of the resistances.

* The resistance of the connecting wires or any other known resistance used in the first case to weaken the stronger electromotive force should of course be subtracted from B.

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If the batteries which are compared are very dissimilar in electromotive force, a British Association apparatus will generally be most convenient; but it is frequently of interest, as in the example below, to offset two batteries of nearly equal power, in which case a box of coils will be found much more exact.* Assuming the electromotive force of a Daniell's battery of two cells to have been measured, that of a Bunsen cell, usually a little inferior, is thus incidentally determined, for from Ohm's Law we have,

$$E = c(B + r)$$

and since $e = cr$, we have

$$\frac{E}{e} = \frac{B + r}{r} = \frac{B}{r} + 1 = \frac{R}{r' - r} + 1,$$

from which the value of E may be calculated when e is known, and vice versa.

*The tenths of ohms are estimated by the usual method of interpolation noticing the relative amounts of the galvanometer deflection in two cases.

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Example.

2 Daniell's cells neutralize current from 1 Bunsen with a common resistance of . . . . . 28.0 ohms.

The common resistance producing equilibrium is . . . . . 168.0 ohms,

When the additional resistance in Daniell's circuit is . . . . . 10.0 ohms.

Hence the internal resistance of the Daniell's battery,

$$B = \frac{28 \times 10}{168 - 28} = \quad . \quad . \quad 2.00 \text{ ohms.}$$

Average resistance of 1 cell, 1 ohm $= 0.99 \times 10^9$

Compare result of Ohm's Method . 1.03×10^9

" " " Mance's " . 0.96×10^9

Electromotive force of Daniell's battery from Ohm's Method,

1.20 + 1.16 volts. = . . 2.36×10^8

Electromotive force of Bunsen cell,

$$2.36 \times 10^8 \times \frac{28}{28 + 2} = \quad . \quad 2.20 \times 10^8$$

Values tabulated . . . . 1.67 to 2.00×10^8

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EARTH-INDUCTOR.

[Reserved in 1884–85.]

The Absolute Measurement of Resistance and Electromotive Force.

I. A large coil of N turns of wire, enclosing as many areas whose average value is S, is made to revolve continuously about a vertical axis, registering by means of a chronograph the time, t, of each quarter-revolution. A commutator is provided so as to turn all the induced currents in the same direction through a galvanometer whose reduction factor is $\frac{H}{K}$, causing a mean deflection, A, not perceptibly affected by the rapid oscillations of the current.

If the horizontal intensity of the earth's magnetism be H, the electromotive force in the circuit is given by the equation

$$E = \frac{HNS}{t},$$

and the total resistance of the circuit, R, is determined in absolute measure by the formula,

$$R = \frac{E}{C} = \frac{HNS}{t} \div \frac{H}{X} \text{ tangent } A = \frac{NSX}{t \text{ tangent } A},$$

where X is the constant of the galvanometer.

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It is of interest to determine the resistance, R, by means of a Wheatstone's Bridge, in terms also of the unit established by heating water, since, if the mechanical equivalent be rightly chosen, these two measures should agree.

II. Substituting, if necessary, a much more sensitive galvanometer, a sufficient velocity may be given to the earth-inductor to neutralize, very exactly, a small opposing electromotive force, as that, for instance, of a thermoelectric junction, which is thus determined in absolute measure* by the first formula, understanding, of course, by t, the new time of a quarter-revolution.

For, by definition, the unit current in a unit length of wire, creates a unit field at unit distance, that is, it acts upon a unit magnetic pole at unit distance with the unit of force. The direction of this action is perpendicular to the wire and to the line joining it with the magnetic pole. The reaction, which must be equal and opposite, is due to the unit magnetic field, which, by definition, is created by the unit of magnetism at unit distance; therefore the unit of current, flowing through the unit length of wire, acts and is acted upon with

* With an earth-inductor of a not inordinate number of turns of wire, the electromotive force of a Daniell's or Clark's cell might be thus directly determined.

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unit force in this unit field, and a current, C, will under the same conditions be acted upon in a field, H, with CH units of force. To move a small length of wire, l, with the velocity, v, against this force for the unit of time, requires an amount of work, W, equal to $CHlv$. But since the wire and its motion are at right angles to the lines of force, it cuts a number, n, of them in one second, equal to Hlv; hence $W = Cn$. An equivalent amount of energy must be expended somewhere by C units of electricity; each unit must therefore do n units of work more than it otherwise would; hence n measures, by definition, the electromotive force due to this portion of the wire.

The same is evidently true whatever change may be made in the direction of the motion or of the wire, since we need consider only the components in the original direction. The total electromotive force in any circuit is therefore equal to the total number of lines of force cut per second; and since HNS lines are cut by the earth-inductor in each quarter-revolution, we have

$$E = \frac{HNS}{t},$$

from which the second formula obviously follows.

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CLARK'S POTENTIOMETER.

[Reserved in 1884–85.]

The Comparison of Two Electromotive Forces by means of Clark's Potentiometer.

Four points are chosen suitably situated in a chain of resistances. The positive and negative poles of the weaker battery or source of electromotive force are connected through a galvanometer with A and B, those of the stronger source through a second galvanometer to A and C, while an auxiliary battery, more powerful than either, is connected with A and D, so as to send a current in series through A, B, C, and D.

When both galvanometers show no deflection, the electromotive forces neutralized by this single current are obviously to each other as the resistances AB and AC, the differences of potential of the points A, B, and C, being in this case proportional to the resistances between them.

The wire of a Wheatstone's apparatus will generally supply the necessary resistance, AC, CD being an additional length of German silver

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wire interposed in the auxiliary circuit so as to neutralize the greater electromotive force, and B being the position of the contact-piece at which the current from the weaker source is reduced to zero.

But when the electromotive forces are very unequal, it is better to substitute two sets of coils with the necessary fixed and variable resistances, adding to AB what is taken from BC, and vice versa, or if AB and BD are first adjusted, adding to BC what is taken from CD, so that in either case the first adjustment may not be disturbed in making the second.

The process might be somewhat simplified if the earth-inductor were chosen as the weaker source of electromotive force, by regulating the velocity so as to reduce the current to zero with any convenient resistance AB. An interesting application would be the determination of the electromotive force of a constant cell like Latimer Clark's* in absolute measure, so as afterwards to serve as a convenient standard for comparison.

* Containing zinc and mercury in a paste of sulphate of mercury and sulphate of zinc, and having an electromotive force estimated at 145.7 million absolute units.

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POGGENDORFF'S METHOD.

The Absolute Determination of Electromotive Force by Poggendorff's Method.

The current from an auxiliary battery of sufficient power, measured by a tangent galvanometer, flows through a uniform straight wire of known linear resistance. A side circuit is formed, containing a delicate galvanometer and the battery whose electromotive force is to be determined, between one end of the wire, where similar poles meet, and a sliding contact-piece, placed by trial so that the current in this branch is reduced to zero.

If the deflection of the tangent galvanometer be A, and the reduction factor $\frac{H}{X}$, the main current, C, must be $\frac{H}{X}$ *tangent* A. If the resistance of a centimetre of the wire be r, the total resistance, R, of the portion of the wire whose length is d will be equal to rd. Hence by Ohm's Law, the difference of potential of the two extremities of this portion is CR, and must evidently be equal to the electromotive force, E, of the battery in question, given accordingly by the equation

$$E = \frac{H}{X} \text{ tangent } A \times rd.$$

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Example.

I. Resistance of Bridge Wire, x, measured by a second Wheatstone's apparatus . . . . 1.869 ohms.

Ditto in absolute measure,

$1.869 \times .989 \times 10^9 =$ . . 1.848×10^9

Length of wire . . . . . 100 cm.

Absolute resistance per cm. . . 0.01848×10^9

Distance of contact-piece from end . 88.25 cm.

Deflection of galvanometer (1 Bunsen cell) . . . . . $51°.1$

Tangent of $51°.1$ 1.239

Reduction factor 0.0564

Electromotive force, 1 Daniell

$1.239 \times .0564 \times 18{,}480{,}000 \times 88.25 = 1.14 \times 10^8$

II. (*Extra.*) Deflection, 2 Bunsen cells . $39°.7$

Distance of contact-piece . . . . 97.5 cm.

Reduction factor, double arc 0.1128

Electromotive force, 1 Leclanché, same way,

$.830 \times .1128 \times 18{,}480{,}000 \times 97.5 = 1.69 \times 10^8$

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THERMOELECTRIC JUNCTION.

[Not required in 1884-85.]

The Determination of Thermoelectric Force by means of a Differential Galvanometer.

A thermoelectric junction is heated in a rubber tube by steam, and a second junction, entirely similar, is cooled by melting ice. The current generated is led through one half of a differential galvanometer, while the other half is connected with a constant battery of known electromotive force, E, and internal resistance, B, so as to act oppositely upon the needle. By means of a box of coils, a resistance, R, is introduced into the battery circuit so that the galvanometer shows no deflection. Then by Ohm's Law, if T be the resistance of the thermoelectric pair and connecting wires, and G that of either half of the galvanometer, we have for the electromotive force, e, the equation,

$$e = \frac{T + G}{B + R + G} \times E.$$

Thermoelectric force is nearly proportional to small differences in temperature; and, being once determined, enables us, with a delicate galvanometer and a thermopile having a sufficient number of junctions, to measure very small differences of temperature with great precision.

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EXAMPLE.

Resistance of half galvanometer .	5.00 ohms.
" " junctions and wires .	1.85 ohms.
Total	6.85 ohms.
Resistance of Daniell's cell . . .	1 ohm.
Additional resistance	2620 ohms.
Half galvanometer	5 ohms.
Total	2626 ohms.
Electromotive force of 1 Daniell, from Poggendorff's method .	1.14×10^8
Thermoelectric force for 100°, $1.14 \times 10^8 \times \frac{6.85}{2626} =$ . .	2.97×10^5
Ditto for 1°	29.7×10^2
Compare with Everett's Units, page 151, Thermoelectric Force for Iron and German Silver for 1° difference of temperature, 1175 + 1750 =	29.34×10^2

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THE CONDENSER.

[Reserved in 1884–85.]

The Absolute Determination of the Capacity of a Condenser.

The condenser is regularly charged and discharged, by means of a suitable commutator, n times in a second, and the *quasi* resistance, r, of the combination is determined by a Wheatstone's Bridge in absolute units. The capacity, c, is then given by the formula,

$$c = \frac{1}{2nr}.$$

For, an electromotive force, e, will by the definition of capacity charge the condenser with a quantity of electricity, ec; hence when the poles are reversed, ec units will be required to neutralize this charge, and ec more units to recharge the condenser in the opposite way; therefore $2ec$ units pass in and out of the condenser at each reversal of the commutator, which therefore transmits $2ecn$ units per second; but since the conden-

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ser with its commutator is equivalent to the resistance, r, the actual current, by Ohm's Law, must be equal to $e \div r$; hence

$$2ecn = \frac{e}{r}, \text{ or } c = \frac{1}{2nr}, \text{ Q.E.D.}$$

The speed of the commutator must lie within a certain limit, in order that the condenser may be fully charged. This condition is known to be fulfilled when, with an increased velocity, the same measure of the capacity is obtained.

Example.

Number of reversals in one second . . 50.
Apparent resistance . . . . . 30,000 × 10.
Capacity, $\frac{1}{8}$ "microfarad"

$$= \frac{1}{2 \times 50 \times 30{,}000 \times 10^9} = \quad . \quad 0.333 \times 10^{-15}$$

Ditto with 100 reversals per second,
same way . . . . . 0.332×10^{-15}

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THE ELECTROMETER.

[Reserved in 1884-85.]

The Determination of Difference of Potential in Electrostatic Measure by Thomson's Absolute Electrometer.

A disc of area, A, is attracted by a large parallel plate at the distance, L, with a force which may be counterpoised by the weight of m grammes in the field of the earth's gravity, g. The difference of potential, E, is then given in electrostatic units by the equation,

$$E = L\sqrt{\frac{8\pi mg}{A}}.$$

The object of the experiment is to compare the electrostatic and electromagnetic units of electromotive force. The former has been found, in the C.G.S. system, to be the larger by about thirty thousand million times, the same number expressing the velocity of light in centimetres per second, and underlying all the relations between the electrostatic and electromagnetic systems of measurement.

The formula is easily established. The perpendicular attraction of any portion of the plate for the unit of electricity at any point of the disc is

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the same as the direct attraction of a portion of a sphere of unit radius centred at that point, with the same electric charge per unit of surface, and subtending at the point the same solid angle; for the quantities of electricity are proportional to the squares of the distances, and inversely to the cosine of the obliquity of the plane surface; while the component of the attraction diminishes with the square of the distance and is proportional to the cosine of the obliquity.

The field of force near the plate is therefore equal to the product of the electrical surface density, d, by the area, 2π, of a hemisphere whose radius is unity. The disc, being oppositely charged, also creates a field, $2\pi d$, so that the resultant is $4\pi d$, and the work, E, necessary to transfer the unit of electricity from one to the other, must be $4\pi Ld$.

The attraction, mg, felt by the disc of area, A, in the field, $2\pi d$, due to the plate, is evidently equal to $2\pi d \times Ad = 2\pi Ad^2$; hence we have

$$d = \sqrt{\frac{mg}{2\pi A}}, \text{ and substituting,}$$

$$E = 4\pi Ld = 4\pi L\sqrt{\frac{mg}{2\pi A}} = L\sqrt{\frac{8\pi mg}{A}}, \text{ Q.E.D.}$$

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NUMBERS FREQUENTLY REQUIRED IN CALCULATION.

Ratio of circumference to diameter . .	3.1416
Latitude of Physical Laboratory .	42° 22½′ N.
Acceleration of gravity	980.37 $\frac{\text{cm.}}{\text{sec.}}$
Pressure of atmosphere at 76 cm.	1.0130 $\frac{\text{megadynes.}}{\text{sq. cm.}}$
Density of dry air at 0° . . .	0.0012924 $\frac{\text{g.}}{\text{cu. cm.}}$
Velocity of sound in ditto	33.22 $\frac{\text{metres.}}{\text{sec.}}$
Specific heat of air, under constant pressure	0.2383
Ratio of ditto to that under constant volume	1.408
Coefficient of expansion of air . . .	0.00367
Mechanical equivalent of heat	41.66 $\frac{\text{megergs.}}{\text{gramme-degree.}}$
Density of hydrogen at 0° . .	0.00008962 $\frac{\text{g.}}{\text{cu. cm.}}$
Hydrogen set free by unit current	0.0001052 $\frac{\text{g.}}{\text{sec.}}$

	Density.	Specific Heat.	Coefficient of Expansion of Volume.
Iron or Steel	7.0 to 7.9	.11	.000033 to 44
Copper, Brass or German Silver	7.8 to 8.5	.094 to .095	.000052 to 57
Glass, Crown and Flint	2.5 to 4.4	.19?	.00002 to 3

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